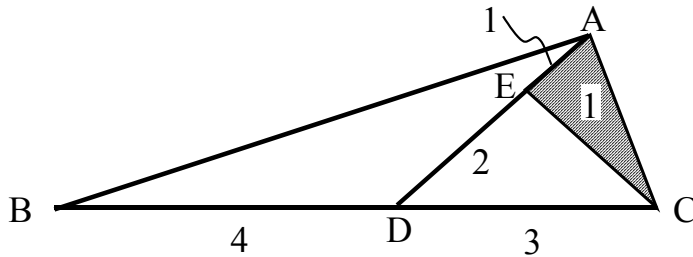1. Label points D,E as shown. This question makes use of the fact that: "If two triangles have the same height, their areas are proportional to their bases".
   ΔAEC has area 1
   ΔADC, with same height, 3 times the base, has area 3
   ΔABC, with same height as ΔACD (now looking at the <u>vertical</u> height), and 7/3 times the base length, has area 7.
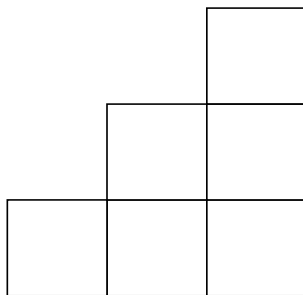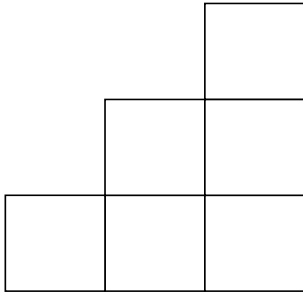

   Answer: <u>7</u>

2. This question was actually really easy. Just count how many faces you see.
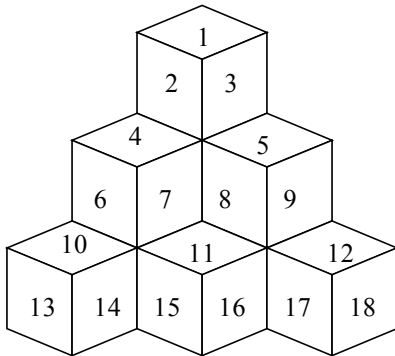
   And both back sides, and the bottom look like

   So have surface area 6 each. So the total surface area is 18 + 6 + 6 + 6 = 36.
   Note: You can think of the object looking like

from 6 different directions, giving answer 6 × 6 = 36.

Answer: <u>36</u>

3. You almost certainly must use variables for this one. Let x be the current of the river. So when the sailor sails upstream, his speed is 9-x. When he sails downstream, his speed is 9+x. Let D be the distance between the two houses. Using Distance = Speed × Time

   Time Spent Going To Friend's House = 2 × (Time Spent Coming Home)

   D / (9 - x) = 2 × D / (9 + x)

   1 / (9 – x) = 2 / ((9 + x)

   9 + x = 18 – 2 x

   3 x = 9

   x = 3

Answer: <u>3</u>

4. You really just have to use trial and error

Price	Cups Sold	Money Received	Cost Spent	Profit
$1.00	30	$30.00	$15.00	$15.00
$1.10	28	$30.80	$14.00	$16.80
$1.20	26	$31.20	$13.00	$18.20
$1.30	24	$31.20	$12.00	$19.20
$1.40	22	$30.80	$11.00	$19.80
$1.50	20	$30.00	$10.00	$20.00
$1.60	18	$28.80	$9.00	$19.80

The profit is decreasing now, so we conclude that $1.50 is the best price, i.e., raise the price by 50 cents. Note the student did not have to try as many prices as we did to get the right answer. Note: To really understand what is going on, like why the profit goes up and then down <u>forever</u>, you need to model the problem with a mathematical shape called a parabola, that looks like
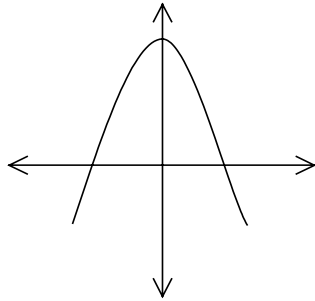
The point of this question was just to test the student's intuition.

Answer: <u>Raise by 50 cents (or raise to $1.50)</u>

5. Once again, a speed question involving variables. We can solve with variables by letting D represent the "size" of the wall, and then
   o The painter's speed is D / 2
   o His brother's speed is D / 3
   o Together they wood need
      $D / (D/2 + D/3) = 1 / (1/2 + 1/3) = 1 / (5/6) = 6/5$ hours or 72 minutes.

   If you are unsatisfied by the wall having a "size", this problem actually has an easy intuitive explanation:
   His brother is 2/3 as useful as him, since the brother spends 3 hours to do something he can do in 2 hours, so together they count as 5/3 of himself working. So he needs 3/5 the time to finish. 3/5 (2 hours) = 6/5 hours.

   Answer: <u>6/5 hours or 72 minutes</u>

6. The last digit follows a pattern:

Number	Last Digit
77	7
$77^2 = 5929$	9
$77^3 = 456533$	3
$77^4 = 35153041$	1
77^5	7
77^6	9
77^7	3
77^8	1
77^9	7

since when a number ends in 7, multiplying it by a number that ends in 7 (77 in this case) gives you a number that ends in 9 (since $7 \times 7 = 49$). If a number ends in 9, multiplying it by a number that ends in 7 gives you a number that ends in 3 (since $7 \times 9 = 63$). Either way, students hopefully can deduce that pattern 7, 9, 3, 1, 7, 9, 3, 1, 7, … So 77^1 has last digit 7
77^5 has last has last digit 7
$77^9, 77^{13}, \cdots 77^{77}$ have last digit 7
(all these exponents leave remainder 1 when divided by 4) So 77^{79} has last digit 3.

Answer: <u>3</u>

7. The question is easiest solved with a Venn diagram:
   A represents the people who worked <u>ONLY</u> on Chapter 1.
   D represents the people who worked <u>ONLY</u> on Chapter 2.
   F represents the people who worked <u>ONLY</u> on Chapter 3.
   B represents the people who worked on Chapters 1 and 2, but <u>NOT</u> 3.
   C represents the people who worked on Chapters 1 and 3 but <u>NOT</u> 2.
   E represents the people who worked on Chapters 2 and 3 but <u>NOT</u> 1.
   G represents the people who worked on <u>ALL</u> 3 Chapters.
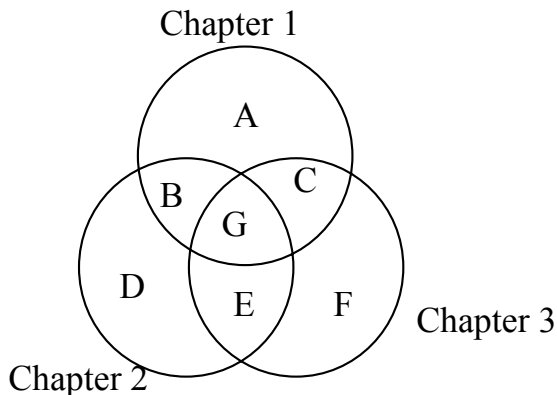
$A + B + C + G = 26$ (1)
$D + B + E + G = 26$ (2)
$F + C + E + G = 26$ (3)
$B + G = 7$ (4)
$C + G = 7$ (5)
$E + G = 7$ (6)
$A + B + C + D + E + F + G = 60$ (7)
There are many ways to work out G, the easiest probably being
$(1) - (4)$: $A + C = 19$
$(2) - (6)$: $D + B = 19$
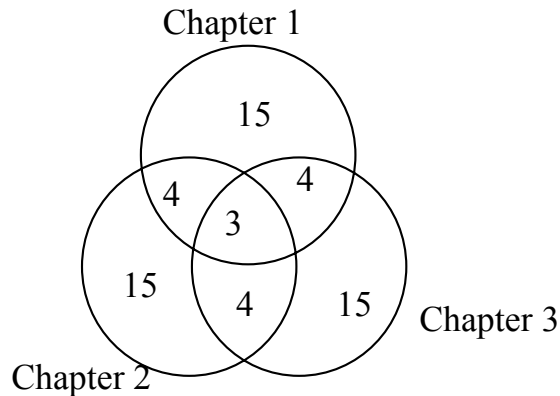$(3) - (5)$: $F + E = 19$
So $A + B + C + D + E + F = 57$
Since $A + B + C + D + E + F + G = 60$ by (7), $G = 3$.

Note: There are lots of symmetries in this question, so the answer is not as hard to get as it looks. Some students may intelligently "guess" the answer correctly without any equations.
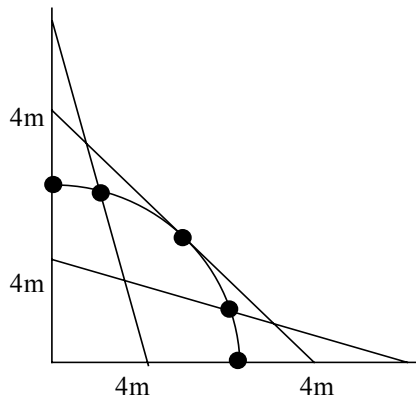
Note 2: If you want to know all the numbers, here they are:

Chapter 1

15

4   4

3

15

4   15   Chapter 3

Chapter 2

(You can calculate them with the equations)

Answer: 3

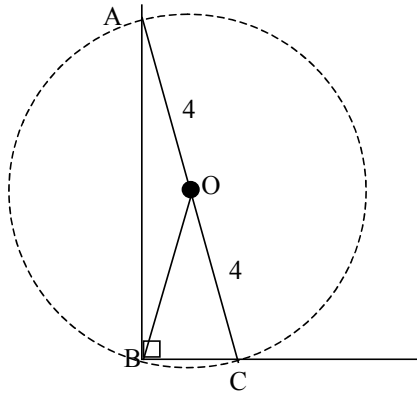8. The path the cat takes is a part of a circle!

4m

4m

4m   4m

You can draw many lines to convince yourself, or try it with a real object in the dark with a glow-in-the-dark middle point! So the answer is $2\pi r/4$, since it is a quarter the circumference of a circle with radius 4. So the answer is $2\pi(4)/4 = 2\pi = 2 (3.141\ldots) \approx 6.28$ m $= 628$ cm.

Note: This question was difficult. It is possible for students to "guess" that the shape is a circle. To give a mathematical "proof", we use some advanced geometry:

At every position of the ladder, consider triangle ABC as drawn. If we draw circle through A, B, C, AC is the diameter, since $\angle$ABC is a right angle (this is a fact in geometry). So the position of the cat O is the centre of this circle of radius 4. So OB has length 4, <u>ALWAYS</u>, regardless of the position of the ladder. So the path of O is <u>always</u> 4 m away from B, hence it forms a circle around B.

Do not worry if the proof was too complicated. This was supposed to be an interesting and difficult last question for the students to test their creativity.

Answer: <u>628 cm</u>