

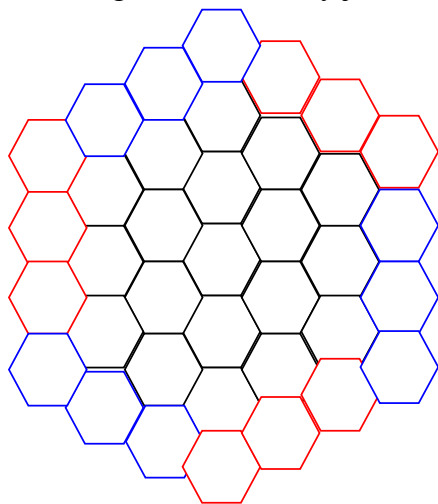
1. The answer is not 50%, since after removing the first sock, the chances of removing a second one of the same colour is lowered. After removing the first sock, only 4 of the remaining 9 socks have the same colour, so the probability is $\frac{4}{9}$. Note: Since there are an equal amount of red socks and blue socks, we do not care what colour the socks removed are.

Answer: $\frac{4}{9}$

2. We want a number less than 50, with a remainder of 4 when divided by 7, and a remainder of 2 when divided by 5. Trial and error can help you figure out that this number has to be 32 (go through the numbers that leave a remainder of 4 when divided by 7, that is, 4, 11, 18, 25, 32, 39, 46 and see that only 32 works). 32 divided by 9 is 3 remainder 5. Note: Trial and error can be avoided through more advanced algebraic methods; we will not discuss them here since they are more complicated.

Answer: 5

3. You can get the answer by just really drawing it out:



Or you can notice that there will be 3 in each “row” (shown by colours), by noticing the pattern. Either way the answer is $3 \times 6 = 18$.

Answer: 18

4. $(25 + 2) + (23 + 2) + (21 + 2) + \dots + (3 + 2) + 1 = (27 + 25 + 23 + \dots + 5) + 1$ minutes. How do we calculate the term in brackets? Once again, some students may just punch every term into their calculator. However, we can again use an “averaging” trick. It is not hard to count there are $(27-5)/2 + 1 = 12$ numbers to add. The average of these numbers is $(27 + 5) / 2 = 16$. So the final answer is $16 \times 12 + 1 = 192 + 1 = 193$ minutes.

Answer: 193

5. 12 pigeons is not enough, since 4 can go on each boat. However, it is not hard to see that 13 pigeons smallest number that can guarantee a boat with 5 pigeons, since each boat only holds 4 otherwise. Note: This question was based off a mathematical law called the Pigeonhole Principle.

Answer: 13

6. The tricky part is figuring out what the radius is. In fact it does not matter, because we are looking at a ratio. The student could have gotten the answer by giving the sphere any radius, but we will show our calculations with radius as the variable r . Surface area of whole sphere is $4\pi r^2$, Surface area of half sphere is $4\pi r^2 / 2 + \pi r^2$, since πr^2 is the area of the inner circle that is “exposed”. So the answer is $4\pi r^2 / (4\pi r^2 / 2 + \pi r^2) = 4\pi r^2 / 3\pi r^2 = 4/3$.

Answer: 4/3

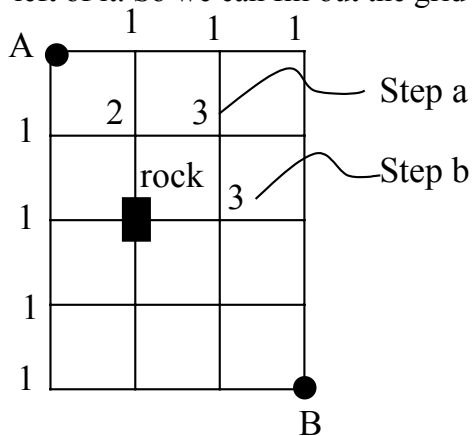
7. Many students can get the answer by just drawing all possible paths. We will show 2 different approaches.

Method 1: The number of ways to arrive at each point is just the sum of:

(1) the ways to arrive at the point above it

(2) the ways to arrive at the point left of it

because to get to that point, we have to first step on the point above it or the point left of it. So we can fill out the grid as follows:



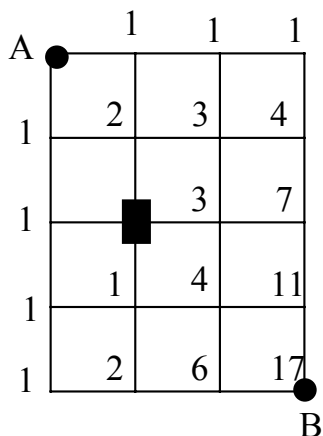
step (1) The edges are filled with “1” since there is only 1 way to get to them.

Step (2) this point is filled with $1 + 1 = 2$

Step (a) this point is filled with $2 + 1 = 3$

Step (b) this point is filled with $0 + 3 = 3$ (treat rock as 0; 0 ways to step on rock)

... and so on, until we fill out the entire grid:



So the answer is 17.

Method 2: If you know

$$\binom{k}{n}$$

or “n choose k” means, you can solve the problem as follows:

The number of paths from A to B, ignoring the rock, is

$$\binom{7}{3}$$

(choose 3 out of 3 “right” paths out of 7).

$$\binom{7}{3} = \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 35$$

However, we need to subtract the paths that do go through the rock, since we counted them in the first number. This number is

$$\binom{3}{1} \binom{4}{2} = 3 \times 6 = 18$$

since $\binom{3}{1}$ is the number of paths from A to the rock, $\binom{4}{2}$ is the number of paths from the rock to B. So the answer is $35 - 18 = 17$.

Answer: 17

8. Really we just need to check all of the numbers less than 25. There are many tricks we can use to speed up the checking, like skipping prime numbers, since their only factor is 1 (other than itself). You will find the only ones are 12, 18, 20, and 24. so the answer is 4.

Answer: 4