Student Name:_____

Please write your name on *every* page.

Section F

F1.

Timmy is taking a round trip drive from Waterloo to Toronto. When traveling from Waterloo to Toronto, Timmy drives at an average speed of 80km/h. When driving back from Toronto to Waterloo, Timmy drives at an average speed of 120km/h. What was Timmy's average speed throughout the entire trip?

Answer to F1: _____

F2.

Let x and y be positive integer numbers such that $x + y = 12$. Find the maximum possible value of $x^2 + 4xy + y^2$.

Answer to F2: _____

F3

Consider that the product of the non-zero digits of 2024 are equal to 16, a perfect square. We call any number with this property "quirky". How many positive quirky numbers there are less than 2024?

Answer to F3: _____

F4

Alice and Bob are playing a game involving a standard deck of cards and n dice. Each "turn", Alice draws a random card from the deck, while Bob rolls the dice. If Bob's dice roll produces a number higher than the number on Alice's randomly drawn card, Bob wins. Otherwise, Alice shuffles the card back into her deck, and Bob moves on to the next dice. If Bob runs out of dice, he loses the game. Find the minimum value of n such that Bob is expected to win at least 90% of the time.
(For the sake of this question, A = 1, J = 11, Q = 12, and K= 13 in the deck of cards)

Answer to F4: _____

Student Name:_____

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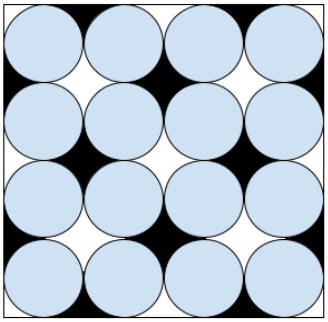
F5

Consider a chessboard pattern constructed by alternating colouring gaps between circles of radius 1 in a larger square. One such example of a pattern with 16 circles is provided



Consider a construction of this pattern with 100 circles and find the area of the black coloured region.

Answer to F5: _____

F6

In the milky way, there exists the planet Polaris, where all inhabitants come from two groups. The righteous Verits always tell the truth, while the evil Mendas always lie. You are given a collection of 10 inhabitants from the planet, all of which are either Verits or Mendas. You can ask any of these aliens any question that results in a yes or no answer. You are tasked with identifying which of the two groups each alien in the collection belongs to. Your predecessor was not very good at his job, and ended up taking 29 questions to identify the aliens. In how many fewer questions can you use that will guarantee that you identify all of the aliens?

Answer to F6: _____

Student Name:_____

Please write your name on *every* page.

F7

Note that a polynomial is a mathematical expression containing variables and integer coefficients. For example, $x^2 + 3x + 7$ and $2x^4 + 7x + 1$ are examples of polynomials. For any given polynomial, we say its degree is equal to the highest exponent on any variable in the polynomial (the examples above have degree 2 and 4 respectively). Consider a polynomial with degree 2024. If you substitute $x = 0$, this polynomial is evaluated to 6. If you substitute $x = 1$ into the polynomial, you will get a non-zero value. The polynomial has only integer roots. There are two possible values for the coefficient of the x^{2022} term. Find the difference between them

Answer to F7: _____

F8

Consider the base 2024 number 20242024 ... 2024 (where the number is 2024 repeated 2024 times). Find the remainder of this number when it is divided by 17.

Answer to F8: _____