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6 Section F

F1

A math professor curves his class according to a *square root curve*: he takes a student's raw score of $X\%$ on an exam, and curves it to $10\sqrt{X}\%$ (so that 0% goes to 0% and 100% goes to 100%). If the difference between a student's score before and after the curve is 16%, and the student scored at least 50% before the curve, what was their original score?

Answer to F1: _____

F2

In a classroom, if all boys get into groups of 2 and all girls get into groups of 3, there are 32 total groups. If instead all boys get into groups of 5 and all girls get into groups of 2, there are 26 total groups. How many students are in the class?

Answer to F2: _____

F3

A degree 3 polynomial $x^3 + bx^2 + cx + d$ is such that b and c are integers between 0 and 9, and such that $x = 10$ is a root. What is the largest possible value of $|d|$?

Answer to F3: _____

F4

Initially, there are two amoebas in Amoebaland. At the end of each day (starting with day 1), each amoeba tries to divide into more amoebas. For each amoeba, its success rates are as follows:

- With probability $\frac{1}{4}$, it fails (no new amoeba are born)
- With probability $\frac{1}{2}$, it divides into two amoeba (so one new amoeba is born)
- With probability $\frac{1}{4}$, it divides into three amoeba (so two new amoeba are born)

What is the expected number of amoeba in Amoebaland at the beginning of day 3?

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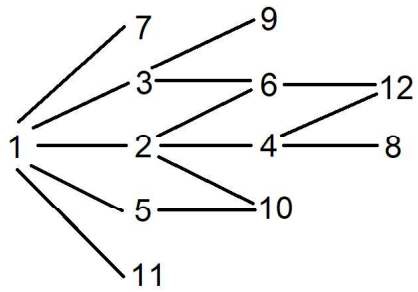
Answer to F4: _____

F5

We say that x is a proper divisor of y if y is divisible by x and $x \neq y$. A tree of the numbers from 1 to n is constructed by drawing a line between x and y precisely when x is a proper divisor of y, and there is no z such that x is a proper divisor of z and z is a proper divisor of y. For example, here is the tree for numbers 1 to 12.



How many lines does the tree for numbers 1 to 50 contain?

Answer to F5: _____

F6

Three numbers are randomly and independently chosen from the unit interval (between 0 and 1). What is the probability their median is less than $\frac{3}{4}$ of their mean?

Answer to F6: _____

F7

Let $\varphi(n)$ denote the number of positive integers $m \leq n$ for which $\gcd(m, n) = 1$ (that is, m and n share no common factors other than 1). For k a positive integer, let $\varphi_k(n)$ denote $\varphi(\varphi(\cdots(\varphi(n))\cdots))$ where the function φ is applied k times. For example, $\varphi(10) = 4$ since 1, 3, 7, and 9 are the only numbers below 10 which have no factors in common with 10 other than 1, while $\varphi_2(10) = \varphi(\varphi(10)) = \varphi(4) = 2$, since 1 and 3 are the only numbers below 4 which have no factors in common with 4 other than 1, and $\varphi_3(10) = \varphi(2) = 1$, since 1 is the only factor less than or equal to 2 which has no factors in common with 2 other than 1 itself.

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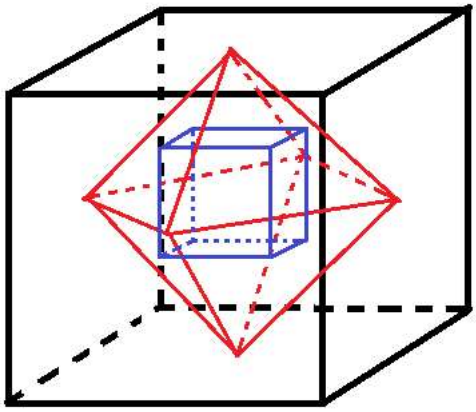
What is the largest k such that $\varphi_k(1000) \neq 1$?

Answer to F7: _____

F8

A regular octahedron is inscribed inside a cube, with each vertex of the octahedron being the centre of one of the cube's faces. A smaller cube is inscribed inside the octahedron, with each vertex of the smaller cube being the centre of one of the octahedron's faces. How many times larger is the volume of the large cube compared to the volume of the small cube?



Answer to F8: _____