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4 Section D

D1

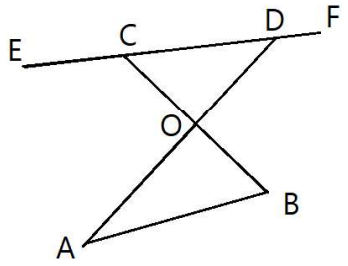
A kilogram is 2.2 pounds. If someone weighs 190 pounds, how many kilograms do they weigh?

Solution. The decimal number 2.2 is equal to the fraction $\frac{11}{5}$. Then $190 \div \frac{11}{5} = \frac{190 \times 5}{11} = \frac{950}{11}$ kg.

Answer to D1: $\frac{950}{11}$

D2

In the diagram below, $\angle \mathrm{OAB} = 30^\circ$ and $\angle \mathrm{OBA} = 50^\circ$. What is the sum of $\angle \mathrm{OCE}$ and $\angle \mathrm{ODF}$?



Solution. If $\angle \mathrm{OAB} = 30^\circ$ and $\angle \mathrm{OBA} = 50^\circ$, then $\angle \mathrm{AOB} = 180 - 50 - 30 = 100^\circ$. By the X-pattern rule, $\angle \mathrm{COD} = \angle \mathrm{AOB} = 100^\circ$. Then $\angle \mathrm{OCD} + \angle \mathrm{ODC} = 180 - 100 = 80^\circ$. Then $\angle \mathrm{OCE} = 180^\circ - \angle \mathrm{OCD}$ and $\angle \mathrm{ODF} = 180^\circ - \angle \mathrm{ODC}$, so that $\angle \mathrm{OCE} + \angle \mathrm{ODF} = (180^\circ - \angle \mathrm{OCD}) + (180^\circ - \angle \mathrm{ODC}) = 360^\circ - (\angle \mathrm{OCD} + \angle \mathrm{ODC}) = 360 - 80 = 280^\circ$.

Answer to D2: 280°

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D3
How many prime numbers are there less than 50?

Solution. We can find prime numbers by trying to divide them by other numbers that aren't 1 or themselves. Doing this, we get that $2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43$, and 47 are the primes less than 50. There are 15 of them.

Answer to D3: 15

D4
What is the value of $4 + \frac{1}{2+\frac{1}{1+\frac{1}{3}}}$?

Solution. We have that $4 + \frac{1}{2+\frac{1}{1+\frac{1}{3}}} = 4 + \frac{1}{2+\frac{1}{\frac{4}{3}}} = 4 + \frac{1}{2+\frac{3}{4}} = 4 + \frac{1}{\frac{11}{4}} = 4 + \frac{4}{11} = \frac{48}{11}$.

Answer to D4: $\frac{48}{11}$

D5
If the greatest common divisor of two numbers is 10 and their least common multiple is 100, what is the product of the two numbers?

Solution. If two numbers have a greatest common divisor of 10, they are both multiples of 10. If their least common multiple is 100, they are both divisible by 100. The factors of 100 between 10 and 100 are $10, 20, 25, 50$, and 100. We have that the least common multiple of 10 and 100 is 100, and their greatest common divisor is 10. Then $10 \times 100 = 1000$. Note that 20 and 50 are also two numbers that satisfy these requirements.

Answer to D5: 1000

D6
If four fair coins are tossed, what is the probability of getting an odd number of heads?

Solution. The probability of getting 1 head is the same as the probability of getting 3 heads (treat the 1 tail you get as a head and the heads as tails), so we only need to find the former. Outcomes that could give us 1 head are HTTT, THTT, TTHT, and TTTH. This is 4 outcomes, out of $2^4 = 16$ possibilities, so the probability is $\frac{4}{16} = \frac{1}{4}$. Then the probability of getting an odd number of heads is $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$.

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Answer to D6: $\frac{1}{2}$

D7

How many numbers between 1 and 100 are multiples of 2 but not multiples of 3?

Solution. Multiples of 2 from 1 to 100 are the even numbers, so $2, 4, 6, \cdots, 100$. This is exactly half of these numbers, so 50. Multiples of 2 that are also multiples of 3 are multiples of 6. Since $6 \times 16 = 96 < 100$ but $6 \times 17 = 102 > 100$, there are 16 multiples of 6. Then there are $50 - 16 = 34$ multiples of 2 that are not multiples of 3.

Answer to D7: 34

D8

Liam and Patrick are playing a game. Liam picks a number between 1 and 2023. Patrick then repeatedly attempts to guess Liam's number. After each guess, Liam tells Patrick whether the guess was too low, too high, or exactly his number (in this last case the game ends). What is the minimum number of guesses that Patrick has to make in order to guess Liam's number correctly regardless of which one he chose?

Solution. With each guess, Patrick can eliminate at most half of the numbers plus one from consideration. This is because is only told if the guess is too high, too low, or just right. If he eliminated more than this number in one scenario, then in the opposite scenario he would eliminate less than this number and the game would be harder to win. His strategy in the worst case proceeds as follows

Guess 1: Patrick guesses $\frac{2023-1}{2} = 1011$. In the worst case scenario, he is wrong, and so he eliminates his guess, plus either the 1011 higher or lower numbers. Thus 1011 numbers remain.

Guess 2: Patrick guesses the middle of the 1011 numbers, and eliminates 506 of them so that 505 remain.

Guess 3: Patrick guesses the middle of 505 numbers, eliminates 253 of them, so that 252 remain.

Guess 4: Patrick guesses either of the two middle numbers of the 252 left. The side with one less number than the other gets eliminated, along with the guess itself, so 126 numbers remain.

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In subsequent rounds, the number of remaining numbers similarly becomes 63, then 31, then 15, then 7, then 3, then 1. On his final guess, Patrick is now guaranteed to be correct. This is his 11th guess.

Answer to D8: 11