Student Name: ______________________________

Please write your name on *every* page.

3 Section C

C1

A sequence has terms beginning with $1, 3, 5, \ldots$, and each term is 2 larger than the term before it. What is the 10th term in the sequence?

Solution. Counting up by twos, we get $1, 3, 5, 7, 9, 11, 13, 15, 17, 19$, and so 19 is the 10th term in the sequence.

Answer to C1: 19

C2

In a video game, there are 8 worlds. Each world has 10 levels, and each level has 3 stars to collect. What is the minimum number of worlds you need to have visited in order to collect at least 100 stars?

Solution. If we visit 3 worlds, and collect all three stars in each of the 10 levels in each world, we will have $3 \times 10 \times 3 = 90$ stars. So we have to visit a fourth world. This four world has $3 \times 10 = 30$ stars, which is more than the $100 - 90 = 10$ that we still need.

Answer to C2: 4

C3

The following table is a list of buses that someone can take to the mall, with the departure and arrival times.

Departure	Arrival
9:43	10:01
9:37	9:59
9:35	9:54
9:49	10:10

What is the shortest time that a bus takes to reach the mall?

Solution. For the first row, it takes 17 minutes to reach 10 o’clock from $9:43$, and then we wait 1 more minute, so the trip takes 18 minutes. For the second and third rows, the trips take $59-37=22$ and $54-35=19$ minutes respectively. The last row takes 11 minutes to reach 10 o’clock, and then 10 minutes afterwards, for a total of $11+10=21$ minutes. So the shortest trip takes 18 minutes.

Answer to C3: 18minutes

C4
Which two numbers from 1 and 9 (not including 1), when multiplied by themselves, have a product that ends in themselves?

Solution. We have that $2\times 2=4, 3\times 3=9, 4\times 4=16, 5\times 5=25, 6\times 6=36, 7\times 7=49, 8\times 8=64, 9\times 9=81$. We see that 5 and 6 are the numbers that work for this.

Answer to C4: 5 and 6

C5
A magic square has each number from 1 to 9 in a cell exactly once, and every row, column, and diagonal sums to the same value. If

8	1	6
3		
		?

is a magic square that is partially filled in, what number replaces the question mark?

Solution. Each row, column, and diagonal of the magic square has to sum to $8+1+6=15$. Then via the first column, the bottom left square must be $15-8-3=4$. By the right diagonal, the centre square must be $15-4-6=5$. By the left diagonal, the bottom right square must be $15-8-5=2$.

Answer to C5: 2

C6
A weekend day is a Saturday or a Sunday. If the month of December has 31 days, and the first day of the month is a Thursday, how many weekend days are there in December?

Solution. If the first day of the month is a Thursday, the first weekend days happen on the 3rd and 4th of the month. Then the second pair are on the 10th and 11th of the month, the third pair on the 17th and 18th, the fourth pair on the 24th and 25th, and then the 31st and final day of the month is a Saturday. Hence there are 9 total weekend days.

Answer to C6: 9

C7
If 23 is divided by a one digit number, what is the maximum possible product of the quotient and remainder?

Solution. We make a table to outline all of the possibilities.

divisor	quotient	remainder	quotient $\times$ remainder
1	23	0	0
2	11	1	11
3	7	2	14
4	5	3	15
5	4	3	12
6	3	5	15
7	3	2	6
8	2	7	14
9	2	5	10

We see that the maximum is 15.

Answer to C7: 15

C8
Brock and Perrin are playing a game. The expression

$$4 + 3 \times 2 - 4 + 5 \times 6 \div 2 - 1$$

is written on a chalkboard. Brock goes first by placing a left bracket immediately before a number somewhere in the expression, and then Perrin places a right bracket somewhere immediately to the right of a different number somewhere in the expression, and it must be after the left bracket so that the expression makes sense. For example, Brock could place a left bracket before the second 4 to make

$4 + 3 \times 2 - (4 + 5 \times 6 \div 2 - 1$

and then Perrin could place a right bracket after the 6 to make

$4 + 3 \times 2 - (4 + 5 \times 6) \div 2 - 1$

The new expression is then evaluated. In this case, it equals -8.

Brock wants the new expression to be as high as possible and Perrin wants it to be as low as possible. Assuming both players know this and play with perfect strategy, what number does the new expression end up equalling?

Solution. Without any brackets, the expression evaluates to

$$4 + 3 \times 2 - 4 + 5 \times 6 \div 2 - 1 = 20$$

If Brock places the left bracket before the second 2, Perrin is forced to put it after the 1. This new expression evaluates to 36. Notice that if Brock places the left bracket before the first 4, then Perrin can place his after the first 2 to keep the value of the expression at 20. Perrin can similarly make such a choice if Brock places the left bracket before 3 (after 2), or before the second 4, 5, or 6, (after the second 2).

The only case remaining is if Brock places the left bracket before the first 2. Here Brock can place the right bracket after the second 4 to get

$$4 + 3 \times (2 - 4) + 5 \times 6 \div 2 - 1 = 12$$

To prevent Perrin from forcing the value of the expression to be the same or smaller, it is in Brock's best interest to place the left bracket before the second 2, so that the expression evaluates to 36.

Answer to C8: 36