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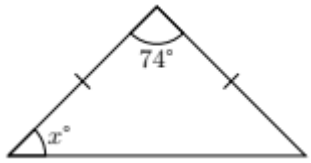
3 Section C

C1
What is the value of angle x° in this diagram?



Solution. Since the triangle is isosceles, the two missing angles are equal. Then $2x = 180 - 74 = 106$ and so $x = 53$.

Answer to C1: 53°

C2
An arithmetic sequence of numbers is as follows: 1, 7, 13, 19, ... where each pair of consecutive numbers has a common difference. What is the 8th number in this sequence?

Solution. The pattern is that we add 6 each time, so then the next terms are $25, 31, 37$, and then the 8th term is 43.

Answer to C2: 43

C3
A 6-sided die is rolled twice. What is the probability of getting the same number on the top face both times (as a reduced fraction)?

Solution. If both dice match, then there are 6 choices for which number they show. In general, there are 6 options for the first die and 6 for the second, and they are independent of one another, so there are $6 \times 6 = 36$ total options. Thus the desired probability is $\frac{6}{36} = \frac{1}{6}$.

Answer to C3: $\frac{1}{6}$

C4
The Fibonacci sequence is the sequence $1, 1, 2, 3, 5, 8, \cdots$ where each term after the first two is the sum of the previous two terms. What is the tenth term in the sequence?

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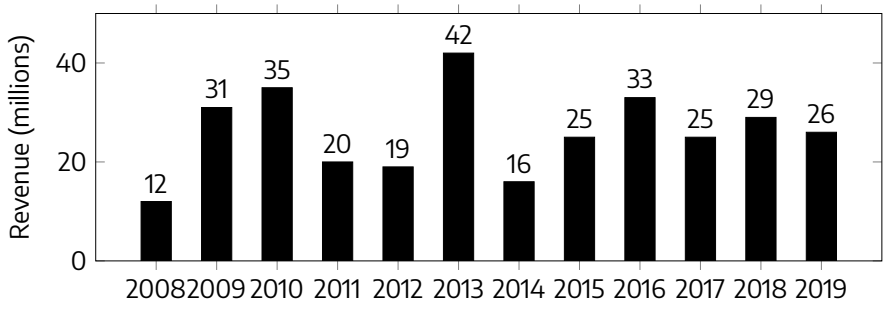
Solution. We are given that term 5 is 5 and term 6 is 8. Thus term 7 is $8 + 5 = 13$, term 8 is $13 + 8 = 21$, term 9 is $21 + 13 = 34$, and term 10 is $34 + 21 = 55$.

Answer to C4: 55

C5

The Dedekind Company produced a graph of its yearly revenues in millions of dollars from 2008 to 2019. The three year mean revenue of a company is the company's average revenue over three consecutive years. From the data below, which three consecutive years from 2008 to 2019 had the highest three year mean revenue? (Write the answer as three consecutive year numbers, separated by commas.)



Solution. The easiest way to compute this is to keep a running total of the last three, and then subtract the previous year and add the next to "move the group of three sum along". In doing this, we find that the years 2016, 2017, 2018 have the highest three year mean revenue.

Answer to C5: 2016, 2017, 2018

C6

A magic square is a grid with the property that every row, column, and main diagonal sums to the same value. Find the value of x in the partially constructed magic square below.

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16	3		13
5		11	x
9		7	12
4	15		

Solution. Based on the first column, the magic constant is $16 + 5 + 9 + 4 = 34$. Then in the third row the remaining value is $34 - 9 - 7 - 12 = 6$. Then in the second column the remaining value is $34 - 6 - 15 - 3 = 10$. Finally, the value of x is $34 - 5 - 11 - 10 = 8$.

Answer to C6: 8

C7

A machine called Algo takes a number as an input and uses a procedure to convert that number into a different number, which the machine produces as output. The procedure is as follows for one conversion:

1. Add 4 to the input number.
2. Multiply the number in the previous step by 2.
3. Take the square root of the number obtained in the previous step. If the result is not an integer, round that number down to the nearest integer.

For instance, Algo will produce 5 as an output if the number 9 was taken as input ($9 \to 13 \to 26 \to 5$). If you let Algo run for more than one conversion, it will take the output from the previous conversion as input for the next conversion (e.g. 9 as input - first conversion: $9 \to 5$; second conversion: $5 \to 4$). If you gave Algo the number 15 as an initial input and let it run for 3 conversions, what will be its final output?

Solution. First pass: $\sqrt{2 \times (15 + 4)} = \sqrt{38}$ rounds down to 6.
Second pass: $\sqrt{2 \times (6 + 4)} = \sqrt{20}$ rounds down to 4.
Third pass: $\sqrt{2 \times (4 + 4)} = \sqrt{16} = 4$.

Answer to C7: 4

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C8

Normally, the expression $5 \times 4 + 3 \times 2 - 1$ is evaluated using the usual order of operations, giving an answer of 25. If instead, the operations can be evaluated in any order, what is the largest possible value of the expression? (for example, if we evaluate operations from left to right, the value becomes $20 + 3 \times 2 - 1 = 23 \times 2 - 1 = 46 - 1 = 45$).

Solution. In general, we want to multiply bigger numbers, rather than adding them, as this will produce a larger result. Thus we want to produce the biggest multiplication possible, so we should aim to do this as late as possible. Subtraction will make things smaller, so we should save that for last. Thus our order becomes addition, then multiplication, then subtraction. This gets us $5 \times 4 + 3 \times 2 - 1 = 5 \times 7 \times 2 - 1 = 70 - 1 = 69$.

Answer to C8: 69