4 Section D

D1

How many two-digit whole numbers have at least one 5? (Two digit whole numbers range from 10 to 99).

Solution. The numbers are 15, 25, 35, 45, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 65, 75, 85, and 95, so the answer is 18. A smarter way to do this is without writing out all the numbers is to notice that there are 9 numbers that end with a 5 (15, 25, ..., 95), and 10 numbers that start with a 5 (50, 51, ..., 59), and one number that starts and ends with a 5 (55), for a total of $9 + 10 - 1 = \boxed{18}$.

Answer to D1: 18

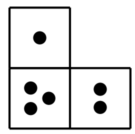
D2

How many digits does 10^{10} have?

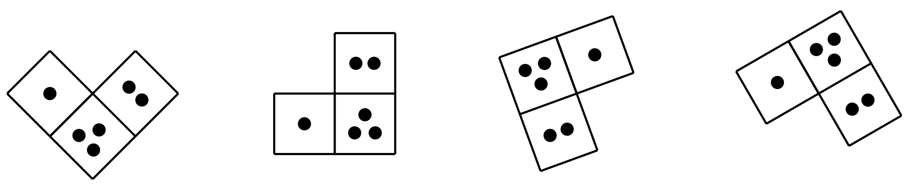
Solution. Multiplying by 10 adds a zero to the end of the number, so $10^{10} = 10000000000$ (one followed by 10 zeros). So 10^{10} has 11 digits.

Answer to D2: 11

D3
This is an L-triomino. It is a labeled block that looks like this:



Three of the following shapes are rotations of this L-triomino, but one of them is not. Write the letter corresponding to the shape that is **not** a rotation of the L-triomino on the answer line.



A B C D

Solution. This problem can be solved by looking at each L-triomino and tilting your head until it has the same position (the L shape) as the example. Doing this for A, B, and C reveals an identical triomino, but if you do this for D, you will get an L-triomino with two dots in the top square and one on the bottom right, so $\boxed{\text{D}}$ is not a rotation.

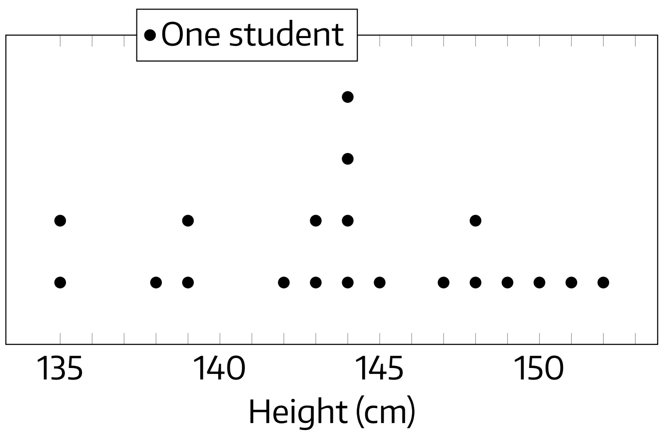
Answer to D3: D

D4
What is the minimum number of fair coins that you must flip to have at least 9/10 probability of getting at least one coin landing on heads?

Solution. If we have at least 9/10 probability of getting heads on at least one toss, then we have at most 1/10 probability of not getting heads on at least one toss (i.e. getting tails on all tosses). On a single toss, we have a 1/2 probability of getting tails. If we toss a coin n times, the probability of getting tails each time is $(1/2)^n$, so we need to find the smallest n which makes this value less than 1/10. If n = 1 then we get 1/2, n = 2 gives 1/4, n = 3 gives 1/8, but n = 4 gives 1/16 which is less. Thus the minimum number of coins we need to toss is $\boxed{4}$.

Answer to D4: 4

D5
Anika conducted a survey of the heights of his classmates. He produced the following dot plot, where each student is represented by a single dot:



What is the average (mean) height of everyone in the class?

Solution. This question calls for a calculator. Counting the dots reveals that there are 20 students in the class. If we read each dot in the data and add their corresponding heights up, then divide by the number of students in the class, we get the mean height, which turns out to be $\boxed{144}$.

Answer to D5: 144

D6
A rectangle has area 16 and its length is 4 times its width. What is the length?

Solution. We can let the length of the rectangle be l and the width be w. If the rectangle has area 16, then lw = 16. If the length is four times the width, then l = 4w, or w = (1/4)l. Subtituting this into the first equation gives l(1/4)l = 16, or l^2 = 64. Since l is a length, it is positive, and so l = 8. Thus the length of the rectangle is $\boxed{8}$.

Answer to D6: 8

D7
Which of the following statements are true for the number 3^{10}? Write the numbers corresponding to all correct statements. For example, if only 1 and 2 were true, write 12. (This is just an example and has no relationship to the answer.)

1. It is a whole number.
2. It is the volume of some cube with whole number side length.
3. The last digit is a 5.
4. It is an even number.
5. It is an odd number.
6. It is a perfect square.

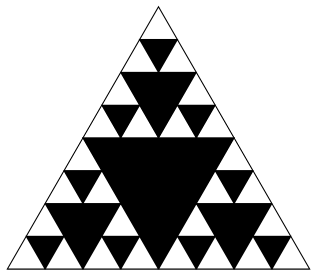
Solution.

1. A whole number to the power of a whole number is a whole number, so 1 is true.
2. A cube with side length l has volume l^3. If $3^1$0 = l^3 then l = $3^($10/3). But 10/3 is not a whole number, and so the side length is not a whole number. Thus 2 is false.
3. If we look at the pattern of the last digit of powers of 3, we see that $3^1 = 3$, $3^2 = 9$, $3^3 = 27$, $3^4 = 81$, and $3^5 = 243$. So the pattern goes 3, 9, 7, 1 and then repeats. In particular, no power of 3 will never end in 5, so 3 is false.
4. Using the powers of 3 pattern we just computed, we can figure out that $3^1$0 ends in 9. Thus it is odd, so 4 is false.
5. If a number is not even, it is odd, so 5 is true.
6. Using the exponent laws, $3^10 = 3^(5 \times 2) = (3^5)^2$, which is a perfect square. Thus 6 is true.

Writing out all of the true statements as a number, we get that the answer is $\boxed{156}$.

Answer to D7: 156

D8
If the area of the smallest shaded triangle ▼ in this picture is 1, what is the entire shaded area?



Solution. We can classify the black triangles in the diagram as being small, medium, or large. The small triangle is known to have area 1. A medium triangle has sides twice as long as the small triangle, and so it will consist of 4 small triangles in total, and thus will have area 4. The large triangle has sides four times as long as the small triangle, so it will consist of 16 triangles total. In terms of shaded triangles, we have 1 large, 3 medium, and 9 small ones. Then the total area is $1(16) + 3(4) + 9(1) = \boxed{37}$.

Answer to D8: 37