Section D

D1
Ciera tosses 2 fair coins. What is the probability that at least one coin lands on heads?

Solution. The probability of getting tails the first toss is $\frac{1}{2}$. Then there is another, independent $\frac{1}{2}$ probability of getting tails the second toss. Hence the total probability of getting both tails is

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$$

Then the probability of getting at least one heads is $1 - \frac{1}{4} = \frac{3}{4}$.

Answer to D1: $\frac{3}{4}$

D2
How many words can you make using all of the letters in MATH? (The words formed do not need to be English words; for example, AHTM is a word.)

Solution. The answer is $4 \times 3 \times 2 \times 1 = 24$. This is because there are 4 choices for the first letter, 3 choices for the second letter (the first letter cannot be reused), 2 choices for the third, and 1 choice for the last.

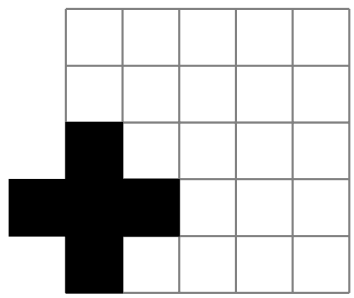
Answer to D2: 24

D3
What is the area of a square whose diagonal has length 4?

Solution. We simply use the Pythagorean Theorem to see that $s^2 + s^2 = 4^2$, so $2s^2 = 16$ and therefore $s^2 = 8$. But then $A = s^2 = 8$, so the area is 8.

Answer to D3: 8

D4
What is the minimum number of crosses required to cover a 5×5 chessboard? Crosses may overlap and may go off the board. The diagram shows a chessboard partially covered by a cross.



Solution. As the diagram's example shows, it's really hard to cover up the corners without "wasting" parts of crosses by letting them go off the chessboard. Thus it is not possible to do in 6 or less. So we need 7 to cover the chessboard.

Answer to D4: 7

D5
Velia rolls a fair 6-sided die 3 times. What is the probability that the total is exactly 10?

Solution. Rolling a six-sided die three times, the following are possible combinations which have a sum of 10:

- 1, 3, 6 (can be rearranged in 6 ways)
- 1, 4, 5 (can be rearranged in 6 ways)
- 2, 3, 5 (can be rearranged in 6 ways)
- 2, 2, 6 (can be rearranged in 3 ways)
- 2, 4, 4 (can be rearranged in 3 ways)
- 3, 3, 4 (can be rearranged in 3 ways)

There are a total of 27 combinations as illustrated above.

Note: Example of rearrangement for 1, 3, 6:

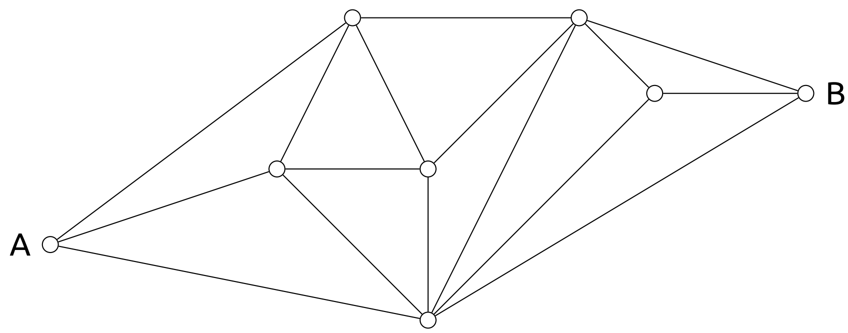
- 1, 3, 6
- 1, 6, 3
- 3, 1, 6
- 3, 6, 1
- 6, 1, 3
- 6, 3, 1

There are a total of $6 \times 6 \times 6 = 216$ possible combinations.

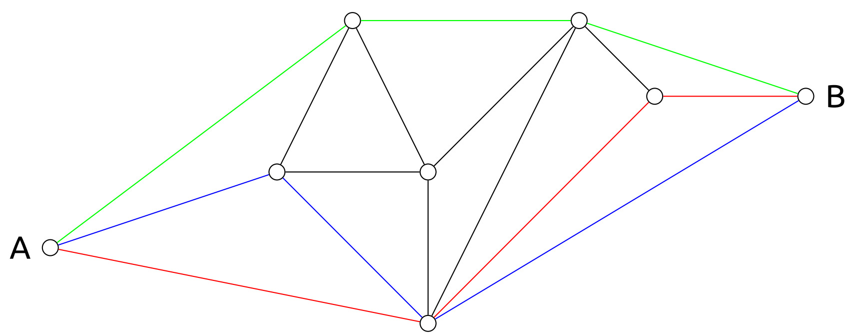
Therefore, the probability is $\frac{27}{216}$, or $\frac{1}{8}$.

Answer to D5: $1/8$

D6
The picture below shows a map of several cities and the roads connecting them. At least how many roads need to be removed to disconnect cities A and B?



Solution.



Note that there are three paths (coloured) from A to city B such that no two distinct paths share a road in common. Thus, in order to disconnect cities A and B by removing roads, at least one road must be removed from each of these three paths. Therefore, the answer is at least 3. On the other hand, removing the roads coming out of city A disconnects A from B, so removing three roads can disconnect cities A and B.

Answer to D6: 3

D7
20 apples weigh more than 25 oranges weigh, but less than 26 oranges weigh. At least how many oranges does it take to weigh more than 13 apples weigh?

Solution. This question can be solved by ratios as follows:

20 apples weighs more than 25 oranges implies 1 apple weighs more than 1.25 oranges 20 apples weighs less than 26 oranges implies 1 apple weighs less than 1.3 oranges

To weigh more than 13 apples, we know the range from above can be derived as follows:

$$13 \text{ apples} \times 1.25 \text{ oranges} = 16.25 \text{ oranges}$$

which means 13 applies would weigh more than 16 oranges, and

$$13 \text{ apples} \times 1.3 \text{ oranges} = 16.9 \text{ oranges}$$

which means 13 apples would weigh less than 17 oranges.

Thus, to weigh more than 13 apples, we would need at least 17 oranges.

Answer to D7: 17

D8
An emirp is a prime number that, when its digits are reversed, produces a different prime number. How many emirps are there between 1 and 100?

Solution. Every single-digit number is not an emirp, since reversing its digits does not give a different number. Also, no two-digit number with tens digit 2, 4, 5, 6, or 8 can be an emirp since reversing its digits would give a composite number. Therefore, we only need to check prime numbers with tens digit 1, 3, 7, or 9.

- Tens digit is 1: 11 is not an emirp since reversing its digits still gives 11, which is not a different number. 13 and 17 are emirps, but 19 is not because $91 = 7 \cdot 13$.
- Tens digit is 3: 31 and 37 are both emirps.
- Tens digit is 7: 71, 73, and 79 are all emirps.
- Tens digit is 9: 97 is an emirp.

Therefore, there are 8 emirps between 1 and 100: 13, 17, 31, 37, 71, 73, 79, and 97.

(Note: Some contestants were mistakenly given an interpretation of the question which removed the condition that reversing the digits of an emirp had give a *different* prime number. Under that interpretation of the question, there are five additional emirps: 2, 3, 5, 7, and 11. Therefore, for the contest both the answers 8 and 13 were accepted.)

Answer to D8: 8