# "The Nine Chapters on the Mathematical Art" Contest (NCC) 2016 © 

Official Solutions

## Section B

## B1

$$
1,2,3,4,1,2,3,4,1,2,3,4, \ldots
$$

If the pattern continues, what is the sum of the first 30 numbers?
Solution. The pattern is repeating. The repeating unit is $1,2,3,4$ and the period is 4 . The first 28 numbers, then, are 7 copies of this repeating unit. So the first 28 numbers sum to $(1+2+3+4) \times 7$. The last two numbers are 1 and 2 .

So we calculate:

$$
(1+2+3+4) \times 7+1+2=10 \times 7+1+2=70+1+2=73
$$

Answer to B1: 73

B2
In simplest form, what is $\frac{2}{5}-\frac{1}{15}$ ?
Solution. We must promote the fractions to a common denominator. 15 is the least common multiple of 5 and 15 , so we can use that. Then

$$
\frac{2}{5}=\frac{2 \times 3}{5 \times 3}=\frac{6}{15}
$$

and hence

$$
\frac{2}{5}-\frac{1}{15}=\frac{6}{15}-\frac{1}{15}=\frac{5}{15}
$$

which we can simplify to

$$
\frac{5}{15}=\frac{5 \div 5}{15 \div 5}=\frac{1}{3}
$$

## B3

Find the area of the shape below:

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Solution. We can split this shape into four squares:


Each square has area $2^{2}=2 \times 2=4$, so the total area is

$$
4 \times 4=16
$$

B4
The picture below shows a map of several cities and the lengths of the roads connecting them. What is the length of the shortest path from city A to city B?


Solution. The shortest path is drawn in bold below:


The length of this path is

$$
2+2+3+3=10
$$

Answer to B4: 10

B5
In how many different positions can a domino be placed on the grid below without overlapping any of the shaded squares?


Solution. All 16 possible domino placements are enumerated in the diagram below.


Answer to B5: 16

B6
Tanya is training for a marathon. She runs 10 km each day from Monday to Friday and 20 km each day during the weekend. If Tanya starts training on a Monday, how many days will it take her to run a total of 500 km ?

Solution. In 7 days, she will have run

$$
5 \times 10 \mathrm{~km}+2 \times 20 \mathrm{~km}=90 \mathrm{~km}
$$

so it should take her at least

$$
500 \mathrm{~km} \div 90 \mathrm{~km}=5
$$

full weeks. But

$$
90 \mathrm{~km} \times 5=450 \mathrm{~km}
$$

is 50 km too short, so she needs to run for five more days, Monday to Friday, to reach 500 km . In total she runs for

$$
7 \times 5+5=35+5=40
$$

days.

Answer to B6: 40

B7
There are 4 people in total. How many ways can you arrange them in a row to take a photo?
Solution. We could name the people A, B, C, and D, and list all 24 ways to arrange them:


Answer to B7: 24

B8
Each node in the tree below has either two or no children. (Children are drawn below their parent are are connected by a line.) Each node is labelled with a positive whole number. Each label is the product of the labels of both its children. No label is 1 . Find the sum of the three missing numbers, which are each marked by question marks (?).

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Solution. There are two ways to fill out the missing numbers:


Either way, the sum of missing numbers does not change; it must be

$$
6+3+2=11
$$

