Part E

1. In the time Alice runs 100m, Bob runs 90m. Thus, Bob's speed is $\frac{9}{10}$ that of Alice's.

   In the time Bob runs 100m, Carol runs 90m. Thus, Carol's speed is $\frac{9}{10}$ that of Bob's.

   Combining these, we get that Carol's speed is $\frac{9}{10} \cdot \frac{9}{10} = \frac{81}{100}$ that of Alice's speed.

   Thus, when Alice finishes running 100m, Carol has run 100m $\times \frac{81}{100} =$ 81m.

   Therefore, when Alice finishes the race, Carol is $\boxed{19\text{m}}$ from the finish line.

2. Note that the only possible way to get a remainder of 98 when we divide by a two-digit number is if the number we're dividing by is 99.

   Let AAB5 be the four-digit number we're looking for. (Here, A, A, B, 5 are the individual digits.) Since AAB5 leaves a remainder of 98 when divided by 99, we must have
   $$\mathsf{AAB5} = 99n + 98$$
   for some positive integer n.

   In particular, the last digit of $99n + 98$ must be 5.

   This means the last digit of $99n$ must be 7, so n must end with a 3.

   We can list all possible values of n and $99n + 98$ in a table to see which one works:

n	3	13	23	33	43	53	63	73	83	93
$99n + 98$	395	1385	2375	3365	4355	5345	6335	7325	8315	9305

   The only four-digit number in the table with the same first two digits is $\boxed{3365}$.

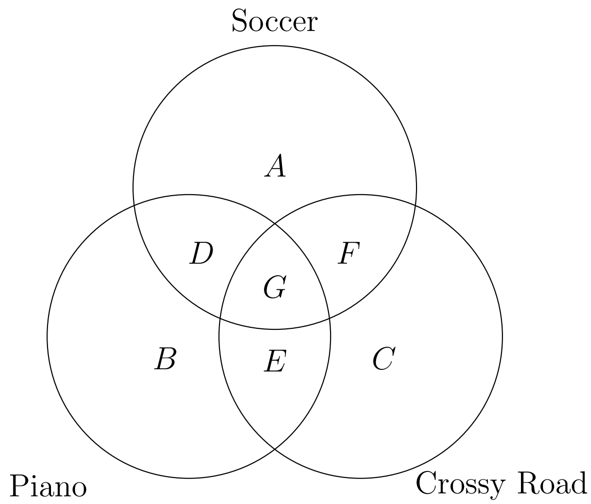
3. Since the paper cup is in the shape of an inverted (upside-down) cone, the water in the cup will form a cone that is similar to the original cone. (Two cones are similar if one is a scaled version of the other.)

   Recall that the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. If we scale the cone by a factor k, then the new radius is kr and the new height is kh, so the new volume becomes $\frac{1}{3}\pi(kr)^2(kh) = \frac{1}{3}k^3\pi r^2 h = k^3 V$.

   This means that when we scale a cone by a factor of k, the volume is scaled by a factor of k^3. For example, if we double each dimension, then the volume is multiplied by 8, and if we halve each dimension, then the volume is multiplied by $\frac{1}{8}$.

   Thus, for the volume of the water to be $\frac{1}{8}$ that of the cup, we need each dimension of the water cone to be equal to $\sqrt[3]{\frac{1}{8}} = \frac{1}{2}$ that of the cup. In particular, the height of the water must be $\boxed{1/2}$ the height of the cup.

4. One way to do this question is to draw a Venn diagram:



We are trying to minimize G (the number of people who play all three things). From the information given in the question, we have the following 4 equations:

$$A + B + C + D + E + F + G = 100 \quad (1)$$
$$A + D + F + G = 90 \quad (2)$$
$$B + D + E + G = 85 \quad (3)$$
$$C + E + F + G = 80 \quad (4)$$

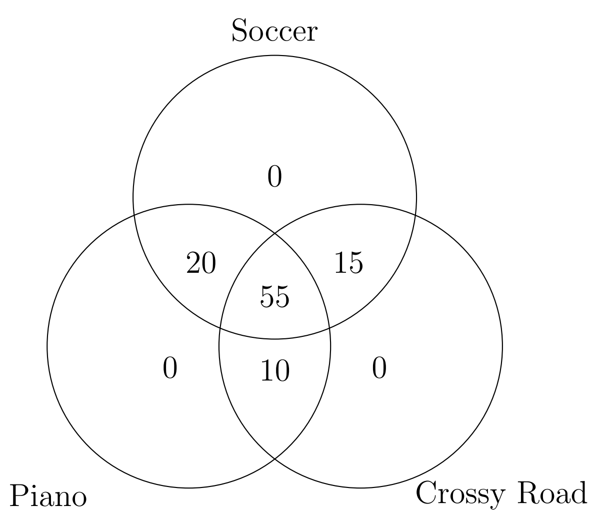
Taking the sum of the last three equations and subtracting twice the first gives

$$G - A - B - C = 55,$$

or

$$G = 55 + A + B + C.$$

Since the variables A, B, C are all non-negative, this implies that $G \geq 55$. Finally, $G = 55$ can be achieved by the following arrangement:



So the answer is $\boxed{55}$.

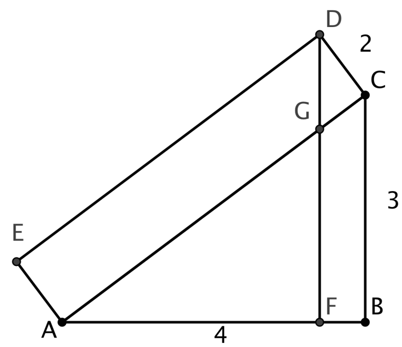
5. Since the die is fair, Stephen rolls each number from 1 to 6 with equal probability 1/6. If Stephen rolls n, then the expected value of his payoff is $\frac{n+1}{2}$. (This is because when a random number is chosen between 1 and n, the average value of that number will be $\frac{n+1}{2}$.)

   So the average number of dollars Stephen will receive for the game is

$$\frac{1}{6}\left(\frac{2}{2}+\frac{3}{2}+\frac{4}{2}+\frac{5}{2}+\frac{6}{2}+\frac{7}{2}\right)=\frac{1}{12}\cdot 27=\frac{9}{4}.$$

6. Observe that $\Delta DCG \sim \Delta ABC$ because their angles are the same:

   - $\angle DCG = \angle ABC$ since they are both 90°
   - $\angle CGD = \angle BCA$ since $\angle CGD = \angle FGA$ (opposite angles) and $\angle FGA = \angle BCA$ (corresponding angles)
   - $\angle GDC = \angle CAB$ since the other two pairs of angles are equal.



   Next, $AC = \sqrt{3^2+4^2} = 5$ by the Pythagorean Theorem.

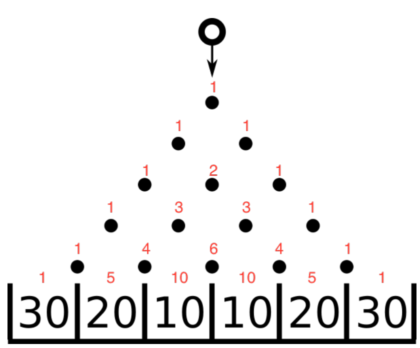
   Finally, using similar triangles, we solve for DG:

$$\frac{DG}{DC}=\frac{AC}{AB} \Longrightarrow \frac{DG}{2}=\frac{5}{4} \Longrightarrow DG=\frac{5}{2}.$$

7. We count the number of possible paths that the ball can take to land in a 10-point bin as follows:

   - Label the topmost peg with 1.
   - For each peg (or bin) on a lower level, the number of ways for the ball to reach the peg (or bin) is equal to the sum of the two numbers immediately above it.

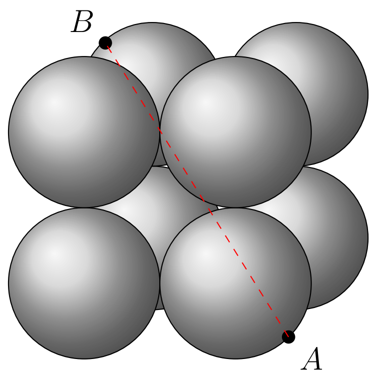
   The labelled diagram is shown below. It is actually just Pascal's Triangle!



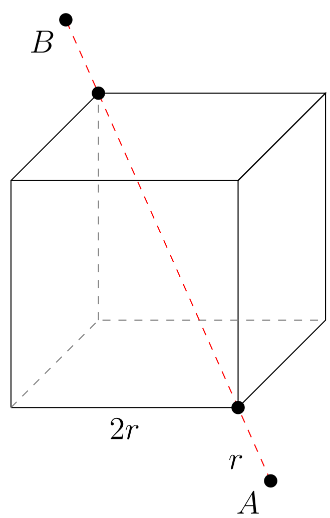
From the diagram, there are 20 ways for the ball to land in a 10-point bin. Each of these 20 paths has a probability of $(\frac{1}{2})^5 = \frac{1}{32}$, so the answer is $\frac{1}{32} \cdot 20 = \boxed{5/8}$.

8. Imagine drawing the smallest sphere that encloses the eight spheres below.

Consider the two points where the bottom-right sphere and the diagonally opposite sphere touch the enclosing sphere (labelled A, B). Then AB is a diameter of the enclosing sphere.



To find the length of AB, consider the diagram below. The vertices of the cube are the centres of the 8 spheres.



If the radius of a small sphere is r, then the side length of the cube is $2r$.

The diagonal of the cube has length $\sqrt{(2r)^2 + (2r)^2 + (2r)^2} = 2\sqrt{3}r$ by the Pythagorean Theorem.

The distance from A to its closest corner is the radius of a sphere, r, and the same for B.

Thus the length AB is $2r + 2\sqrt{3}r$ (which is the diameter of the enclosing sphere). So the radius of the enclosing sphere is $r + \sqrt{3}r$.

Plugging in $r = 1$, we get $\boxed{1 + \sqrt{3}}$.