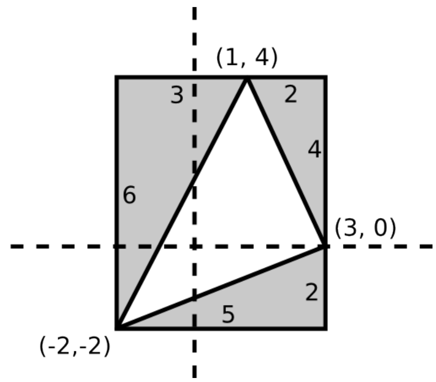
Part F

1. Here is the diagram:



We can get the area of the white triangle by taking the area of the entire rectangle and subtracting the areas of the three shaded triangles.

$$5 \times 6 - \frac{3 \times 6}{2} - \frac{2 \times 4}{2} - \frac{2 \times 5}{2} = 30 - 9 - 4 - 5 = 12.$$

So, the area of the white triangle is $\boxed{12}$.

2. Let t_k denote the number of integers that appear before (and including) the last occurence of the integer k in the sequence. That is, $t_1 = 1, t_2 = 3, t_3 = 6, \ldots$. Notice that $t_k = 1 + 2 + 3 + \cdots + k = \frac{k(k-1)}{2}$.

Let the 2014$^{\text{th}}$ number in the sequence be n. Then $t_{n-1} < 2014 \leq t_n$, which means that

$$\frac{(n-1)(n-2)}{2} < 2014 \leq \frac{n(n-1)}{2},$$

or

$$(n-1)(n-2) < 4028 \leq n(n-1).$$

The only integer n that satisfies this equation (which can be found by trial-and-error) is $\boxed{63}$.

Remark: A closed formula for the nth term in the sequence is $\left\lfloor \sqrt{2n + \frac{1}{4}} + \frac{1}{2} \right\rfloor$.

3. If $(x^2 + x + 47)(x^2 - 4x - 21) = 0$, then either $x^2 + x + 47 = 0$ or $x^2 - 4x - 21 = 0$.

However, $x^2 + x + 47 = 0$ is never true for real numbers, because the quadratic discriminant is negative: $b^2 - 4ac = 1^2 - 4(1)(47) = -187$.

On the other hand, $x^2 - 4x - 21$ has two real solutions, because its discriminant is positive: $b^2 - 4ac = (-4)^2 - 4(1)(-21) = 100$. By Vieta's formulas, the product of the solutions to $x^2 - 4x - 21$ is $c/a = -21/1 = -21$. So, the product of all solutions is $\boxed{-21}$.

Alternatively, after noticing that $x^2 + x + 47$ has no solutions, we can factor $x^2 - 4x - 21$ to obtain $(x-7)(x+3) = 0$. That means the two solutions are $x = 7$ or $x = -3$, and the product of 7 and -3 is $\boxed{-21}$.

4. We manipulate the given equation to something easier to work with. We have

$$\begin{aligned} \frac{1}{x}+\frac{1}{y}&=\frac{1}{4} \\ 4y+4x&=xy \\ xy-4x-4y&=0 \\ xy-4x-4y+16&=16 \\ (x-4)(y-4)&=16 \end{aligned}$$

Since x and y are integers, so are $x-4$ and $y-4$. Hence, $x-4$ and $y-4$ must be two factors of 16 that multiply to 16.

Note that 16 has ten factors (they are ±1, ±2, ±4, ±8, and ±16). For each possible choice of $x-4$ among these ten factors, there is exactly one possible choice for $y-4$ so that $(x-4)(y-4)=16$. Thus, the equation $(x-4)(y-4)=16$ has 10 integer solutions.

Do all the solutions to $(x-4)(y-4)=16$ also satisfy $1/x+1/y=1/4$? The answer is NO, because in the second equation, x and y cannot be 0. If $x=0$ in the first equation, then we have $-4(y-4)=16$, so that $y=0$ (and similarly, if $y=0$ then $x=0$). Thus, $(0,0)$ is the only pair (x,y) satisfying $(x-4)(y-4)=16$ but not $1/x+1/y=1/4$.

Hence there are $10-1=\boxed{9}$ integer solutions to $1/x+1/y=1/4$.

5. We claim that the probability that Bob flips more heads than Alice is *equal* to the probability that Bob flips more tails than Alice. Consider any configuration of coins in which Bob has more heads than Alice. By swapping all the heads to tails and all the tails to heads, we obtain a configuration of coins in which Bob has more tails than Alice. On the other hand, consider a configuration of coins in which Bob has more tails than Alice. By swapping all the tails to heads and all the heads to tails, we obtain a configuration in which Bob has more heads than Alice. Thus, the probability that Bob flips more heads than Alice is equal to the probability that Bob flips more tails than Alice.

Finally, note that since Bob has 11 coins and Alice has 10 coins, it is impossible for them to both flip the same number of heads *and* the same number of tails (because if they did, then that would mean they have the same number of coins). Hence, either Bob flips more heads than Alice or Bob flips more tails than Alice. By our argument above, these two probabilities are equal. Since they are the only possibilities, they must each be equal to $\boxed{1/2}$.

More formally, let H_a and H_b denote the number of heads Alice and Bob flip respectively. Let T_a and T_b denote the number of tails Alice and Bob flip respectively. Then $H_a+T_a=10$ and $H_b+T_b=11$. Let $\Pr(A)$ denote the probability of event A.

Notice that

$$\begin{aligned} \Pr(H_b>H_a)&=\Pr(H_b-1\geq H_a) \\ &=\Pr(10-T_b\geq 10-T_a) \\ &=\Pr(T_b\leq T_a) \\ &=1-\Pr(T_b>T_a). \end{aligned}$$

By symmetry, $\Pr(H_b > H_a) = \Pr(T_b > T_a)$. Hence,

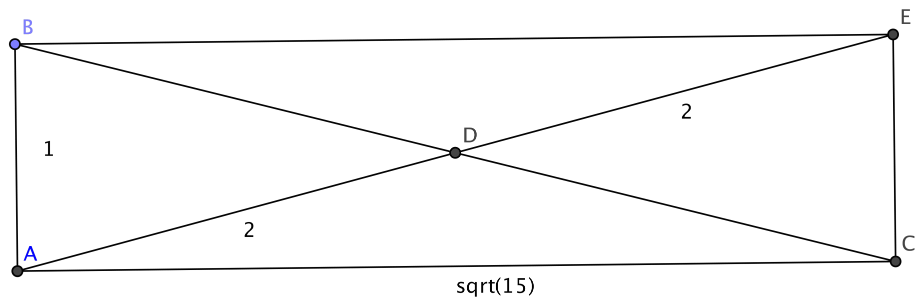
$$1 = \Pr(H_b > H_a) + \Pr(T_b > T_a) = 2\Pr(H_b > H_a),$$

so we have

$$\Pr(H_b > H_a) = \boxed{\frac{1}{2}}.$$

Remark: The same argument shows that whenever Alice and Bob flip an unequal number of coins, the probability that Bob flips more heads (or more tails) than Alice is 1/2.

6. Double the median AD to AE and connect EB and EC, as shown below.



Consider the quadrilateral $ABEC$. Since the diagonals (AE and BC) bisect each other, it follows that $ABEC$ is a parallelogram. Hence, $EC = AB = 1$ and $BE = AC = \sqrt{15}$. Moreover, the triangle ACE (with side lengths 1, $\sqrt{15}$, and 4), is a right triangle since $1^2 + (\sqrt{15})^2 = 4^2$. Thus, $ABEC$ is a rectangle.

Therefore, the area of $\triangle ABC$ is equal to half of the area of rectangle $ABEC$ which is $1 \times \sqrt{15}$. Hence, the area of $\triangle ABC$ is $\boxed{\sqrt{15}/2}$.

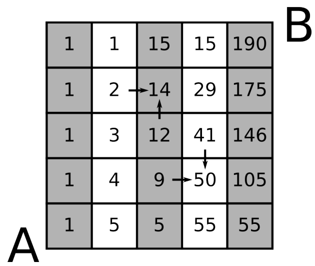
7. Notice that we can fill in the grid with the number of ways to get to each square by starting at A, then filling the columns from left to right, shaded columns upwards and white columns downwards.

We start by putting a 1 in the starting square. Then, we fill in the squares by the following rules:

- The number of ways to get to a shaded square is the sum of the numbers immediately down and to the left.
- The number of ways to get to a white square is the sum of the numbers immediately up and to the left.

This works because the number of ways to get to any square is the sum of the number of ways to get to the squares from which you can enter that square.

This results in the following grid:



The number that ends up the top-right corner is 190, so the number of ways to get to B is $\boxed{190}$.

Remark: This is a technique called *dynamic programming*, and it is very useful.

8. For each clown i, let A_i be the set of days it performs on. Since there are 4 days, A_i is a subset of $\{1, 2, 3, 4\}$. For example, if clown i performs on days 2 and 3, then $A_i = \{2, 3\}$. Note that there are 16 possibilities for A_i, since there are 16 subsets of $\{1, 2, 3, 4\}$ (including the empty subset). Consider the following grouping of the 16 subsets of $\{1, 2, 3, 4\}$:

   $\{\}, \{1\}, \{1, 2\}, \{1, 2, 3\}, \{1, 2, 3, 4\}$

   $\{3\}, \{1, 3\}, \{1, 3, 4\}$

   $\{4\}, \{1, 4\}, \{1, 2, 4\}$

   $\{2\}, \{2, 3\}, \{2, 3, 4\}$

   $\{2, 4\}$

   $\{3, 4\}$

   Now, I claim that two clowns cannot have their A_i's be from the same grouping (which means their A_i's can't be the same). Because if this were the case, then one of them will have their A_i be a (not necessarily strict) subset of the other, which means that clown only performs on days the other also performs on, which means he will never be seen by the other clown.

   Since there are only 6 groupings, there cannot be more than 6 clowns if we want each to have seen the act of each other clown at least once. Now let's show that this is indeed possible with 6 clowns. If $A_1 = \{1, 2\}$ $A_2 = \{1, 3\}$ $A_3 = \{1, 4\}$ $A_4 = \{2, 3\}$ $A_5 = \{2, 4\}$ $A_6 = \{3, 4\}$ then for any pair of them, each one will perform on a day the other doesn't, and therefore each clown will see the act of each other clown at least once over the course of the 4 days. Answer: $\boxed{6}$.