

Part D

1. Note that each girl can "occupy" up to 3 chairs; the one that she sits in and the two on either side of her. On the other hand, each boy occupies 1 chair. Thus, the minimum number of chairs that Jerry could have available to choose from is $50 - (11 \times 3 + 10) = 50 - 43 = \boxed{7}$.

This minimum can be achieved by seating the children such that there are at least 2 empty chairs between each pair of girls.

2. Denote the speed of Grace by v_g and John's speed by v_j . Assume that the distance from home to school is 1 then, $v_g = \frac{1}{30}$ and $v_j = \frac{1}{40}$. The distance that John travelled before Grace started is $d_0 = t_0 \cdot v_j = 5 \cdot \frac{1}{40} = \frac{1}{8}$. If T is the time Grace takes to catch up then $v_g \cdot T$ is the distance that Grace travelled to catch up to John. Therefore,

$$v_g \cdot T = d_0 + v_j T \Rightarrow T = \frac{d_0}{v_g - v_j} = \frac{1/8}{1/30 - 1/40} = 15.$$

So it takes $\boxed{15}$ minutes for Grace to catch up to John.

Remark: In the question, there was ambiguity as to whether the time should be measured from the time Grace left (in which case the answer is 15 minutes), or from the time John left (in which case the answer is 20 minutes). Both answers were marked correct on the contest.

3. First, we compute Alfred's average: $(75 + 83 + 85 + 89)/4 = 83$.

If Bob's average is 3% higher, then Bob must have an 86% average among his 3 tests. To compute what his final test score must be to obtain an 88% average, we can use variables.

Let x represent the 4th test score. Then we have

$$\begin{aligned} 88 &= \frac{86 \times 3 + x}{4} \\ 352 &= 258 + x \\ x &= 94. \end{aligned}$$

So Bob needs a final test score of $\boxed{94}$ % to obtain an average of 88%.

4. First, we determine what combination of digits can sum to 3. There are three possibilities:
 - (a) $1 + 1 + 1 = 3$ (with the other three digits being 0);
 - (b) $1 + 2 = 3$ (with the other four digits being 0) – can be arranged with 1 being first or 2 being first;
 - (c) $3 = 3$ (with the other five digits being 0).

We consider these three cases separately.

Case 1: Three 1's, three 0's.

Note that the first digit must be 1. In the remaining 5 spots, we need to place two 1's and three 0's. There are $\binom{5}{2} = 10$ ways to do this. Alternatively, without "choose" notation, we can count all the possibilities by brute force. Here they are:

111000, 110100, 110010, 110001, 101100, 101010, 101001, 100110, 100101, 100011

Case 2: One 1, one 2, four 0's.

As the first digit must be either 1 or 2, the remaining 5 digits can arrange the unused digit in 5 different ways. This results in 5 ways for each of when the digit starts with 1 or 2 for a total of 10 possibilities. For completeness, here are all the possibilities:

120000, 102000, 100200, 100020, 100002, 210000, 201000, 200100, 200010, 200001

Case 3: One 3, five 0's.

Then the first digit must be 3, and the remaining five digits must be 0. Thus, there is only one combination in this case: 300000.

Hence, the answer is $10 + 10 + 1 = \boxed{21}$.

5. Let x be the amount of milk that 1 brown cow gives per day. Then we have:

$$\begin{aligned} 20 + 15x &= 12 + 20x \\ 5x &= 8 \\ x &= 1.6 \end{aligned}$$

Thus, 1 brown cow gives $\boxed{1.6}$ L of milk per day.

6. The probability that Lenny will take out a red marble first then a blue marble after is $\frac{2}{6} \times \frac{4}{5} = \frac{4}{15}$.

The probability that Lenny will take out a blue marble first then a red marble after is $\frac{4}{6} \times \frac{2}{5} = \frac{4}{15}$.

Thus, the total probability is $4/15 + 4/15 = \boxed{8/15}$.

Alternatively, you can calculate it by

$$\frac{\binom{2}{1} \times \binom{4}{1}}{\binom{6}{2}} = \frac{8}{15},$$

where $\binom{n}{k}$ is the number of ways to choose k things from n things.

7. The surface area of a cube with edge length 3cm is $3\text{cm} \times 3\text{cm} \times 6 = 54\text{cm}^2$. Each cm^2 therefore requires $2/54$ grams of paint. The surface area of a cube with edge length 12cm is $12\text{cm} \times 12\text{cm} \times 6 = 864\text{cm}^2$. Thus, the total paint required is $(2/54) \times 864 = \boxed{32}$ grams.

8. A normal year has 365 days, so between two normal years, the weekday will shift by $365 \bmod 7 = 1$ place. (Here, " $365 \bmod 7$ " means the remainder when 365 is divided by 7.) However, a leap year has 366 days, so if the next year is a leap year, the weekday will shift by $366 \bmod 7 = 2$ places.

For example, consider a year that ends on a Monday. If the next year is a normal year, then it will end on a Tuesday. But if the next year is a leap year, then it will end on a Wednesday.

Consider a period of 40 years with 30 normal years and 10 leap years. Then, in those 40 years, the day of the week will shift by

$$(30 \times 1 + 10 \times 2) \bmod 7 = 50 \bmod 7 = 1.$$

That is, if the first year ends on a Monday, the 40th year will end on a Tuesday. In order to correct this shift of one year, we need exactly one of the years in our 40 year span to be a normal year that would normally be a leap year, that is, a multiple of 100 that is not a multiple of 400. The most recent year for which this works is 1900. The last year in a 40 year period starting with 1900 is 1939, so the answer is 1939.

Notice that this means all years from 1900 to 1938 also work.