

## Part C

1. The average (mean) of numbers is defined as:

$$\frac{\text{sum of \#s}}{\text{quantity of \#s}}$$

In this problem, the fact that the numbers are consecutive odd numbers is a red herring (not needed)—the sum is given as 121, and the number of numbers added together is given as 11. This is all the information that one needs.

The average is  $121 \div 11 = \boxed{11}$ .

2. There are several ways to group the terms to get the correct result. One way is given below:

$$(99 - 97) + (95 - 93) + (91 - 89) + \dots + (3 - 1)$$

Notice that  $99 - 97 = 2$ ,  $95 - 93 = 2$ ,  $\dots$ ,  $3 - 1 = 2$ . Each of the differences has a value of 2.

How many of these differences are in the formula? There are 100 numbers from 1 to 100, but only odd numbers are involved in the formula. The number of odd numbers is one half the number of total numbers. Hence 50 numbers are involved in the formula. But each difference involves two odd numbers. So there are a total of  $50 \div 2 = 25$  differences.

So we have 25 differences, each of which has a value of 2. The entire sum is thus  $25 \times 2 = \boxed{50}$ .

3. Every combo saves exactly \$1 over buying its constituent items individually. For example, buying the fries + soda combo costs \$4, whereas buying them individually would cost a total of \$5.

If all items were bought individually, the total money spent would be  $2 \times \$5 + 2 \times \$3 + 2 \times \$2 = \$20$ . For every combo bought, this cost is reduced by \$1. To reach the minimal price, we need to buy as many combos as possible.

Since each combo is at least 2 items, and we only need 6 items, the maximum possible combos we can buy is  $6 \div 2 = 3$ . It turns out there is a way to buy three combos: buy one of each two-item combo.

So the minimal price is  $\$20 - \$3 = \boxed{\$17}$ , and is attained by buying one of each two-item combo.

4. The most compact (efficient) way of seating is for people to sit with a single seat between them. This avoid leaving large gaps.

Number the seats 1 to 25. Let someone sit on an edge seat, #1. (Leaving that seat empty would be a waste of space.) Someone else will sit at every odd number, leaving even numbers empty. This is the best arrangement. Since there are 13 odd numbers between 1 and 25, the answer to this problem is  $\boxed{13}$  people.

5. If we add 33 and 25 ( $33 + 25 = 58$ ), we will have counted students who have written either quiz. But some students are counted twice—these students have written both quizzes.

We know that the actual number of students who have written at least one quiz is  $53 - 6 = 47$ . So we must have counted  $58 - 47 = 11$  students twice.

Thus  $\boxed{11}$  students have written both quizzes.

6. There are 4 choices for each of 3 digits. The total number of possibilities is  $4^3 = 4 \cdot 4 \cdot 4 = \boxed{64}$ .

7. Since both factories manufactured at a constant rate (same number of bicycles every day), then when combined, bicycles are manufactured at a constant rate.

213 bicycles over 3 days means that there were  $213 \div 3 = 71$  bicycles manufactured per day.

The remainder of this problem may be solved using algebra. Where  $A$  represents the production of Factory A:

$$A + (A + 5) = 71 \implies 2A + 5 = 71 \implies 2A = 66 \implies A = \boxed{33}$$

(Intuition without variables: Suppose Factory B's bicycles are split into two categories. Category I contains the same number of bicycles as Factory A produces. Category II contains 5 bicycles, which is the surplus production by Factory B. If we take Category II away from the total, 71, we get 66. That means Factory A's production combined with of Category I is 66 bicycles per day. But Category I is exactly equal to Factory A's production, so Factory A makes  $66 \div 2 = \boxed{33}$  bicycles per day.)

8. Since Peter thinks his watch is 5 minutes ahead, he will come when his watch shows 1:05. But by then, the time is actually 1:12.

Since Sophie thinks her watch is 7 minutes behind, she will come when her watch shows 0:53. But by then, the time is actually only 0:48.

Peter shows up 12 minutes late and Sophie shows up 12 minutes early, so the difference between their arrival times is  $12 + 12 = \boxed{24}$  minutes.