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1. If $ac + ad + bc + bd = 42$ and $c + d = 3$, what is the value of $a + b + c + d$?

$ac + ad + bc + bd = 42$
$a(c + d) + b(c + d) = 42$
$a(3) + b(3) = 42 \; since \; c + d = 3$
$3(a + b) = 42$
$a + b = 14$
$Thus, a + b + c + d = 14 + 3 = \boxed{\mathbf{17}}$.

2. In multiplying two positive integers X and Y, Mike reversed the digits of the two-digit number X. His erroneous product was 91. What is the correct value of the product of X and Y?

Please note that following additional information was provided during the contest for clarification:

X ≠ 1 and Y ≠ 1

We know 91 can be written as product of 1 and 91, as well as 7 and 13. Since neither X nor Y can be 1, also X is the two-digit number, then X has reversed digits of 13 which is 31 and Y = 7. Therefore, the correct value of the product of X and Y is 31 × 7 = $\boxed{\mathbf{217}}$.

3. How many zeros are at the end of $25!^2$ when evaluated? ($n! = 1 \times 2 \times 3 \times ... \times n$)

First, let's find the number of zeros at the end of 25!. The number of trailing zeros in $25!^2$ will just be twice this number. Fact: the number of zeros at the end of any number is equal to how many factors of 10 it has. Note that $10 = 2 \times 5$. Since 25! has many more factors of 2 than factors of 5, counting its factors of 10 is equivalent to counting its factors of 5. The numbers in the expansion of 25! that contribute a factor of 5 are: 5, 10, 15, 20, and 25. The first four contribute one factor of 5 each, whereas 25 contributes two factors of 5. Thus 25! has $1 + 1 + 1 + 1 + 2 = 6$ trailing zeros. This means there are $6 \times 2 = \boxed{\mathbf{12}}$ zeros at the end of $25!^2$.

4. Two points are randomly chosen on a circle. What is the probability that the distance between them is less than the radius of the circle?

Let O be the center of the circle and call the two points A and B. Point A can be placed anywhere on the circle. Now, where can point B be placed such that $\overline{AB} < r$? To answer this, consider the extreme case: $\overline{AB} = r$. Then, ΔABO will be an equilateral triangle, so $\angle AOB = 60^o$. Thus, if $\overline{AB} = r$, then B must be 60^o from A. Note that B can either be 60^o clockwise or counter clockwise from A. These two possible positions are indicated by B_1 and B_2 in the diagram:

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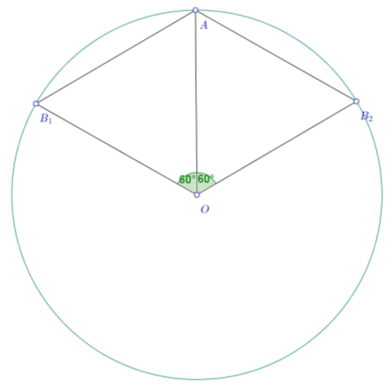

Now it's easy to solve the question. If B lies on the arc B_1AB_2, then $\overline{AB} < r$. Otherwise, $\overline{AB} > r$. The arc B_1AB_2 represents 120^o out of 360^o in the entire circle. Thus the probability is $\frac{120^o}{360^o} = \boxed{\frac{1}{3}}$.

5. A cube has side length 7. Tunnels are dug through the cube in the following way: a square hole of side length 1 is drilled from the centre of each face across to the centre of the opposite face. What is the volume of the resulting solid?

The original volume of the cube is 7^3, or 343. Each tunnel is a rectangular prism with a 1×1 square base and a length of 7 (try to visualize this or draw a 3D diagram on a piece of paper). Thus, the volume of each tunnel is 7. To find the volume of the resulting solid, we start by subtracting the volume of the three tunnels from the volume of the cube: $343 - 3 \times 7 = 322$. But we're not done yet! The $1 \times 1 \times 1$ cube at the centre of the solid is intersected by all three tunnels. When we subtracted the volumes of the three tunnels, we subtracted the volume of the centre cube **3 times**, but we only wanted to subtract it **once**. To compensate, we have to add **twice** the volume of the centre cube to our previous answer, giving $322 + 2 = \boxed{\mathbf{324}}$.

6. Suppose that $4^a = 5,\ 5^b = 6,\ 6^c = 7,\ 7^d = 8$. What is the value of $a \times b \times c \times d$?

Start with $7^d=8$, sub in 6^c for 7, giving $6^{cd}=8$. Then, sub in 5^b for 6, which gives $5^{bcd}=8$. Lastly, sub in 4^a for 5, which results in $4^{abcd}=8$. 4^{abcd} can be written as 2^{2abcd} and 8 can be written as 2^3. Thus, 2abcd=3. The value $a \times b \times c \times d$ is $\boxed{\frac{3}{2}}$.

7. EFCB a quadrilateral inscribed to a circle. Given $\angle BAC = 32°$ and $\angle BEF = 120°$, what is the measure of $\angle CDF$?

Since EFC is an excluded angle of triangle AEF, then $\angle EFC = \angle EAF + \angle AEF$. Given $\angle BAC = \angle EAF = 32°$ and $\angle AEF = 180° - \angle BEF = 180° - 120° = 60°$, $\angle EFC = 32° + 60° = 92°$. Given EFCB a quadrilateral inscribed to a circle, so $\angle EFC + \angle EBC = 180°$.

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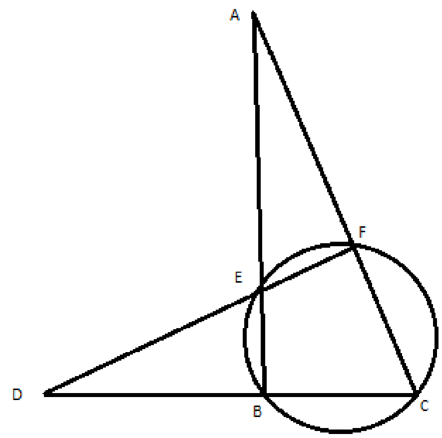
Therefore, $\angle CDF = \angle EBC - \angle DEB = (180° - \angle EFC) - \angle AEF = (180° - 92°) - 60° = \boxed{\mathbf{28°}}$.



8. The 9 reindeer of Santa put their names in a hat and each of them draws one at random, without replacement. Each reindeer grabs the tail of the reindeer it draws (possibly itself). What is the probability the reindeers will end up in a single loop of 9?

We want to count the number of possible cycles one could form with 9 labeled vertices. There are 9! ways of arranging the vertices around the cycle, but notice that each cycle here is counted 9 times, since any cyclic ordering of the 9 letters just represent the same cycle. Thus there are $\frac{9!}{9} = 8!$ total number of cycles of length 9 (i.e. single loops with all reindeers) that can be formed. Note that the total count of all possibilities is 9! since each reindeer is drawing a name from the hat without replacement. Hence, the final answer is $\frac{8!}{9!} = \boxed{\mathbf{\frac{1}{9}}}$.