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1. Let $N = 2^4 \times 3 \times 7 \times \square$. If 360 is a factor of N, what's the minimum number that could be placed in the box?

We begin by finding the prime factorization of 360:

$$360 = 2^3 \times 3^2 \times 5$$

We have

$$N = 2^4 \times 3 \times 7 \times \square$$

To solve $\square$ and have N be a factor of 360, at minimum, the prime factorization of N needs to consist of the prime factors of 360.
From this, we see we need the following additional prime factors: 3×5. Therefore, $\square$ = **15**

2. Let $a\#b = a^2 + ab - b$. For example, $4\#1 = 4^2 + 4 - 1 = 19$. What is 3#(4#5)?

$4\#5 = (4)^2 + (4)(5) - 5 = 31 \Rightarrow 3\#31 = (3)^2 + 3(31) - 31 =$ **71**

3. How many different 4-letter words can be made by rearranging the letters in "POOL" (including POOL)?

There are 4! = 24 rearrangements of a 4-letter. However, since POOL has 2 O's, there are only 12 different ones. One way to see this is pretend we paint one O red, and one O blue. In the list of 24 rearrangements, each word is the same if you switch the red O with the blue O. So each word appears exactly twice. Therefore there are **12** different words.

4. Determine the number of positive integer solutions to the equation $3x + 5y = 80$.

From the equation:

$$3x + 5y = 80$$

We can solve for the number of positive integer solutions by considering possible values for y such that 80 - 5y is divisible by 3. We know y must lie in between 1 and 15 (it cannot equal 16 as 5x16 = 80 and this would result in x = 0, which is not a positive integer).
We first note that if y = 1, 80 - 5y = 75, which is perfectly divisible by 3. Therefore, every increase of y by 3 will result in 80 - 5y to be divisible by 3, as required for 3x to have a positive integer solution (up until the constraint of y = 16).
Note: This pattern also holds if you consider x = 5, then 80 - 3x is divisible by 5 so every increase in x by 5 will also result in 80 - 3x to be divisible by 5 as required for 5y to have a positive integer solution for y (up until constraint of x = 30 since 3 x 30 is greater than 80).

From this, we have the following possible combinations of positive integers such that the equation holds:

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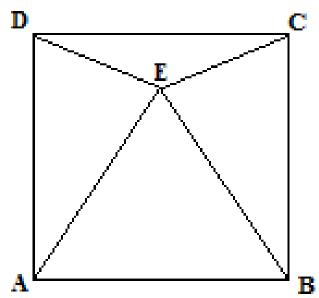
y	x
1	25
4	20
7	15
10	10
13	5

Therefore, there are **5** positive integer solutions to the equation.

5. Point E is chosen inside square ABCD such that ΔABE is equilateral. Find $\angle EDC$.



Since ΔABE is equilateral, we know that its angles are all 60^o. Thus, $\angle DAE = \angle DAB - \angle EAB = 90^o - 60^o = 30^o$. Consider ΔADE. Since AD = AB and AB = AE, it follows that AD = AE. So, ΔADE is isosceles, from which we get that $\angle ADE = \frac{180^o - \angle DAE}{2} = \frac{180^o - 30^o}{2} = 75^o$. Finally, $\angle EDC = \angle ADC - \angle ADE = 90^o - 75^o = \boxed{15^o}$.

6. Sam wrote nine tests, each out of 100. His average on these nine tests is 72%. If his lowest mark is omitted, what is his highest possible resulting average?

The highest possible average of the rest 8 tests is achieved only when the lowest mark omitted is 0. Let μ represent the average. Given the number of scores n = 9 and μ = 72, we can solve the problem by finding the sum of scores as 72 × 9 = 648. If we omit a score of 0, the sum doesn't change but the number of tests is reduced by 1. Thus, using the average formula:

$$\mu = \frac{648}{8}$$
$$\mu = 81$$

Therefore, the highest possible average by omitting the lowest score is **81**.

7. One fair coin is tossed once. If it comes up heads, roll a fair 6-sided die. If it comes up tails, no die is rolled. What is the probability that the die rolls an odd number? (Note that if no die is rolled, then the die rolls a 0.)

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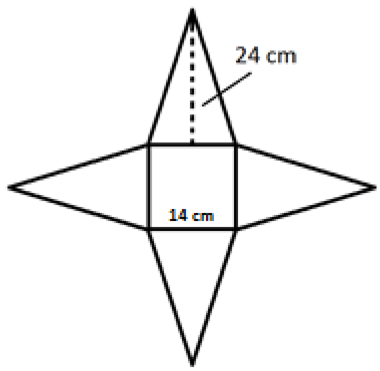
First, the coin flip must turn up heads (since if it turned up tails, then the die wouldn’t even be rolled, let alone roll an odd number). The probability of a coin flip turning up heads is $\frac{1}{2}$. Next, the die must roll an odd number. A die can roll either 1, 2, 3, 4, 5, or 6. Since only three of these are odd, the probability that a die rolls an odd number is $\frac{3}{6}$, or $\frac{1}{2}$. Finally, the total probability is obtained by multiplying the two: $\frac{1}{2} \times \frac{1}{2} = \boxed{\frac{1}{4}}$ or $\boxed{0.25}$ or $\boxed{25\%}$.

8. The net (paper model) of a square based pyramid is shown below. What is **the perimeter of this net** if the square has a side length of 14cm, and all the isosceles triangles have a height of 24cm?



Net

The perimeter of the net is 8 times the length of the side-length of the isosceles triangle (the net is made up of 4 of these triangles). The length of a single side can be obtained with Pythagorean Theorem as we can split the isosceles triangle in half to obtain a right triangle of equal sizes, where a = 24, b = 14/2 = 7 (since it is isosceles).
We solve the following equation:

$$c = \sqrt{24^2 + 7^2}$$

Thus the perimeter of the net is: $8 \times 25 = \boxed{200}$.