Student Name: ______________________________

Please write your name on *every* page.

Section F

F1

Let x be the unique positive real solution to $2^x = \frac{8}{x}$. What is x?

Answer to F1: ____________

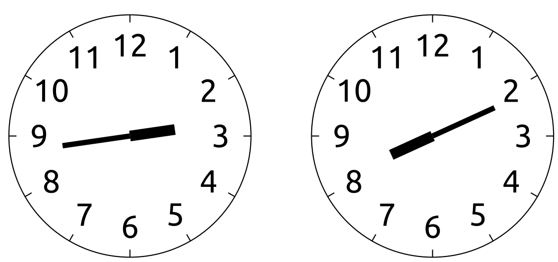
F2

How many words can you make using all of the letters in CANADA? (The words formed do not need to be English words; for example, ACNDAA is a word.)

Answer to F2: ____________

F3

The drawing below shows analog clocks at around 2:44 and 8:11. At these times, the hour and minute hands are pointing in opposite directions. Between 1:00 AM and 11:59 AM on any day, how many times do hour and minute hands point in opposite directions?



Answer to F3: ____________

F4

How many distinct integer solutions (p, q) are there to this equation?

$$p^2 + 3pq + 2q^2 - p - q = 41$$

Answer to F4: ____________

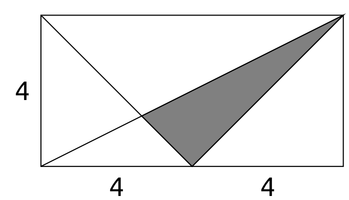
Student Name: ______________________

Please write your name on *every* page.

F5
What is the maximum possible value of $3x - y + z$ if x, y, z are real numbers such that $x \geq 0, y \geq 0, z \geq 0, x + 2y = 5$, and $x + y + z = 7$?

Answer to F5: __________

F6
Find the area of the shaded triangle.



Answer to F6: __________

F7
Define $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. (This is the Fibonacci sequence, and it starts $1, 1, 2, 3, 5, 8, \ldots$.) Find $\gcd(F_{30}, 30)$, where $\gcd(a, b)$ denotes the greatest common divisor of a and b. You may use the fact that $30 = 2 \times 3 \times 5$.

Answer to F7: __________

F8
For all positive integers n, let $f(n)$ be a integer whose value depends on n. Suppose that $f(1) = 1$ and that $f(2n) = 2f(n) - 1$ and $f(2n + 1) = 2f(n) + 1$ for all n. Compute $f(1023)$.

Answer to F8: __________