NCC 2015
Part F

Student name: _____

1. Determine the number which satisfies the following conditions:

   (a) When divided by 4, the remainder is 3;

   (b) When divided by 10, the remainder is 1;

   (c) When divided by 12, the remainder is 3;

   (d) The sum of the quotients from (a), (b), and (c) is 16 more than a third of the number.

   Answer: _____

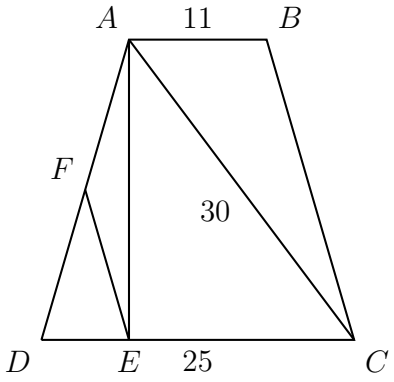
2. Determine the natural number n so that

$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\cdots+\frac{1}{\sqrt{n-1}+\sqrt{n}}=4.$$

   Answer: _____

3. Let $ABCD$ be a trapezoid with $AB = 11$, $CD = 25$, $BC = DA$, and diagonal length $AC = 30$, as shown below. Let E be the foot of the perpendicular from A to CD. Let F be the midpoint of DA. What is length of EF?

   Answer: _____



4. Two squares of a 21×21 square grid are coloured red, while the rest remain white. If two such colourings are defined to be distinct if one cannot be obtained from the other via a series of rotations (about the centre square), how many distinct colourings of this 21×21 square grid are there?

   Answer: _____

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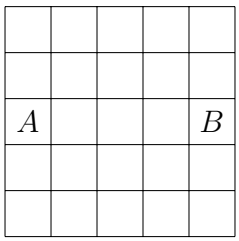
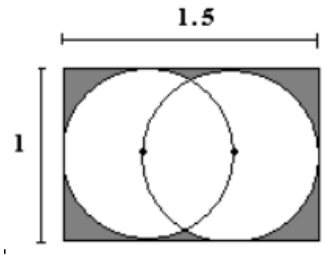
5. How many paths are there from A to B that only move up, down, or right, and never cross themselves?



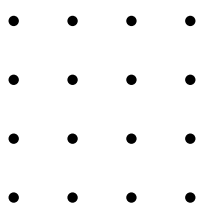
Answer: _____

6. What is the area of the shaded region given that the two identical circles have radius 0.5 and touch the sides of the rectangle (as in the diagram)? Either give an exact answer or round your answer to two decimal places.



Answer: _____

7. Determine the number of triangles with area 1 whose vertices lie on the unit grid shown below:



Answer: _____

8. Consider the following casino game: You roll an 8-sided die (with sides 1 through 8) up to n times. You can stop whenever, and when you do (or finish the nth roll), you get paid in dollars the value of your last roll. Your objective is to maximize the expected (average) amount of money you get paid from playing this game. What is the smallest n such that, under optimal play, you should re-roll even if you get a 7 on the first roll?

Answer: _____