

NCC 2014

Part E

Student name: _____

1. A 3-digit positive integer is called an *elevated number* if:

- (a) Its first digit is strictly less than its second digit, and
- (b) Its second digit is equal to its third digit.

For example, 233 and 477 are elevated numbers, but 132 and 444 are not. How many 3-digit elevated numbers are there?

Answer: _____

2. How many digits does $20^{15} \times 5^{15}$ have?

Answer: _____

3. Without spaces, the positive integers are written out to form a very big number:

1234567891011121314... — What is the digit in the 2014th position?

Answer: _____

4. A rectangular water container has length 10, width 8, and height 6. It is filled with water to a height of 5. A solid cube with side length 5 is slowly lowered into the water (with its bottom face parallel with the base of the container), until just before the water starts flowing out. Find the distance from the top of the cube to the bottom of the water container.

Answer: _____

5. How many non-congruent triangles can be made with side lengths from the numbers $\{1, 2, 3, 4, 2015, 2016\}$ if each number can be used at most once?

Answer: _____

6. On Friday morning, Richard drove from Waterloo to Toronto with an average speed of 80km/h. In the evening, he drove back with an average speed of 120km/h. What is Richard's average speed, in km/h, for the entire trip?

Answer: _____

7. Carl just won \$10,000 and is looking at two ways to save his money and withdraw it in two years. The first option he has is to put the money in a savings account and earn interest at 2% per year. The second option is to invest the money in a fund where there is a 75% probability that he will earn 5% in each of the next 2 years and a 25% probability that the money will lose 5% in each of the next two years. What is the difference in expected payoffs between these two options (i.e. the difference in the amount of how much money he expects under the two options)?

Answer: _____

8. What is the units digit of $33^{77} + 77^{33}$?

Answer: _____