Student Name: ______________________

1. An equilateral triangle of side length 12 is completely filled in by non-overlapping equilateral triangles of side length 1. How many small triangles are required?

Answer: _______________

2. What is the probability that an integer in the set $\{1, 2, 3, \dots 100\}$ is divisible by 2 and not divisible by 3?

Answer: _______________

3. Suppose that the number α satisfies the equation $4 = \alpha + \alpha^{-1}$. What is the value of $\alpha^4 + \alpha^{-4}$?

Answer: _______________

4. Alice flips 8 coins, 3 of which are quarters and 5 of which are pennies. What's the probability that an odd number of pennies come up heads AND the total number of heads (out of all coins) is even?

Answer: _______________

5. The table below shows an example of "Suffix Code". Each letter is represented by a variable length binary sequence "codeword". To send a message, you simply put together the codewords for your word. For example, "DOG" would be encoded as "011011100001". However, decoding a message is tricky because you don't know how the letters are separated. It is called a "Suffix Code" because no codeword is the suffix of another codeword. This property allows for an unambiguous decoding of each message. Please decode the message:

111000101001101011100011000010

character	codeword	character	codeword
!	000	U	01001
B	0100	N	11001
S	01100	C	0101
R	11100	K	1101
E	010	F	00011
D	0110	V	10011
O	1110	I	1011
G	0001	A	111

Answer: _______________

Student Name: ______________________

6. Consider the sequence defined by $a_k = \frac{1}{k^2+k}$ for k ≥ 1.
Given that $a_m + a_{m+1} + ... + a_{2012} = \frac{10}{2013}$ for positive integer m < 2012, find m.

Answer: _______________

7. In trapezoid $ABCD$ with BC parallel to AD, let $BC = 14$ and $AD = 38$. Let $\angle A = 39°$, $\angle D = 51°$, and M and N be the midpoints of BC and AD, respectively. Find the length MN.

Answer: _______________

8. In this problem, we consider the experiment of rolling a set of dice and summing the numbers to get an outcome. Two sets of dice are said to be equivalent if the probability of rolling every outcome is equal in both sets. Consider the following set of dice:

- three 12-sided dice with faces 4, 4, 4, 4, 2, 2, 2, 2, 0, 0, 0, 0
- an 8-sided die

What do the faces of the 8-sided die need to be for this set of dice to be equivalent to the set of three regular 6-sided dice (with faces 1, 2, 3, 4, 5, 6)?

Answer: _______________