Student Name: ______________________

1. Let x be the lowest common multiple of 14 and 24 and y be the greatest common divisor of 14 and 24. Determine the value of x + y.

Prime factorizing 14 and 24, we have that 14 = 2*7, and that 24 = 2^3*3. Thus the lowest common multiple is 2^3*3*7 = 168 and the greatest common divisor is 2 = 2. Thus x + y = 168 + 2 = 170

2. Let $a\Delta b$ refer to $a^2 + ab + b^2$. For example $1\Delta 1 = 1^2 + 1 \times 1 + 1^2 = 3$. Determine the value of $(2\Delta 3)\Delta 4$.

We have that $(2\Delta 3)\Delta 4 = (2^2 + 2 \times 3 + 3^2)\Delta 4 = 19\Delta 4 = 19^2 + 19 \times 4 + 4^2 = 453$

3. Which digits X and Y make 123XY divisible by both 8 and 9?

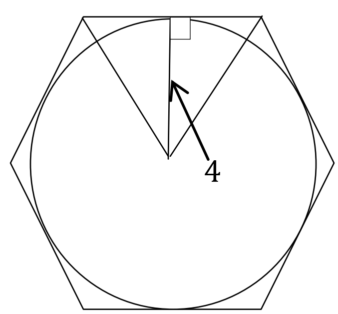
There was a correction for this question that asked to find all possible digits X and Y that make 123XY divisible by 8 and 9. Since 123XY should be divisible by 8, we must have the last three digits 3XY divisible by 8. Also since 123XY is divisible by 9, we must have that 1 + 2 + 3 + X + Y is divisible by 9. Thus X + Y = 3 or 12. Thus the only such numbers are 12384 and 12312, and so the only possibilities for (X, Y) are (1, 2) and (8, 4).

4. In the sequence 32, 8, ______, ______, *x*, each term after the second is the average of the two terms immediately before it. What is the value of *x*?

Continuing the sequence we have: 32, 8, $\frac{32+8}{2} = 20, \frac{20+8}{2} = 14, x = \frac{20+14}{2} = 17$. Thus *x* = 17.

5. A circle of radius 4 is inscribed in a regular hexagon. What is the area of the hexagon?

Draw a line from the centre of the circle to one of the sides of the hexagon and to the two adjacent vertices of the hexagon to create an equilateral triangle (since the interior angles are 60° in a hexagon):



Let *x* represent the side length of the hexagon. Thus by Pythagoras' theorem we get that

Student Name: ______________________

$x^2 = 4^2 + \left(\frac{x}{2}\right)^2$. Thus we get that $x = \frac{8}{\sqrt{3}}$, and so the area of the triangle is $\frac{4\times\frac{8}{\sqrt{3}}}{2} = \frac{16}{\sqrt{3}}$. Since there are 6 of these triangles make up the hexagon, we get that the area of the hexagon is $6 \times \frac{16}{\sqrt{3}} = 32\sqrt{3}$.

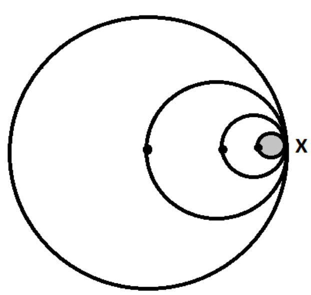
6. Three unique numbers are chosen randomly from 1 to 20 (inclusive). What is the probability that all three numbers are divisible by 5?

There are four numbers amongst the numbers from 1 to 20 that are divisible by 5, namely 5, 10, 15, 20. Since there are $\binom{4}{3} = 4$ ways of choosing 3 numbers that are divisible by 5 from the four that are available and since there are $\binom{20}{3} = 1140$ ways of choosing 3 unique numbers from 1 to 20, thus the probability that all the three chosen numbers are divisible by 5 is $\frac{\binom{4}{3}}{\binom{20}{3}} = \frac{4}{1140} = \frac{1}{285}$.

7. A number is said to be *cool* if it is odd and if the sum of its digits is even. How many *cool* numbers are there between 1 and 100 inclusive?

The numbers between 1 and 100 (excluding 100) can be written in the form XY, where X and Y are digits (notice that here X may be 0, since then XY would represent a one digit number). If XY is *cool*, then X + Y is even, and Y must be either 1, 3, 5, 7 or 9 (since XY has to be odd). Thus X has to be odd (since the sum of two odd numbers is even), meaning that X is either 1, 3, 5, 6 or 9. Since there are 5 possibilities for both X and Y, there are a total of 25 possible *cool* numbers XY. Since 100 is not *cool*, there are 25 *cool* numbers between 1 and 100 inclusive.

8. Four circles are tangent to one another. The circles are placed such that they all touch "X" (point of tangency). Also, each circle touches the center of the one outside it. If the radius of the biggest circle is 24, what is the circumference of the smallest circle?



Let r_1, r_2, r_3 and r_4 represent the radii of the 4 circles from largest to smallest respectively. Since the smaller circles go pass through the centre of the circle one bigger than it, we have that $r_1 = 2r_2 = 2(2r_3) = 2\big(2(2r_4)\big) = 8r_4$. Since we are given that $r_1 = 24$, we get that $r_4 = 3$, and thus the circumference of the smallest circle is $2\pi r_4 = 6\pi$.