"The Nine Chapters on the Mathematical Art" Contest (NCC) 2012©

## Student Name:

1. In a triangle $\mathrm{ABC}, \mathrm{AB}=0.75 \mathrm{~m}$ and $\mathrm{BC}=3.20 \mathrm{~m}$. Find the length of the third side given that it is a whole number of metres.

By triangle's inequality we know that $\mathrm{AB}+\mathrm{BC}>\mathrm{AC}$, and $\mathrm{AC}+\mathrm{AB}>\mathrm{BC}$. Thus we have that $2.45 \mathrm{~m}<\mathrm{AC}<3.95 \mathrm{~m}$. Thus the only possible whole number of meters for AC is 3 m .
2. If six campers can dig six holes in six hours, then how many hours does it take twelve campers to dig twelve holes? (Assume that each camper digs at the same rate.)

Since six campers can dig six holes in six hours, that means the six campers have a digging speed of 1 hole per hour. Thus 12 campers have a digging speed of 2 holes per hour. So 12 campers can dig 12 holes in 6 hours.
3. Flora and Jeremy can build a house together in 2 hours. If Flora can build the same house alone using only 4 more hours, how long would it take Jeremy to build the same house if he worked alone?

Since Flora can build the house in 6 hours, and since she only needs 2 hours to complete it with Jeremy, she contributed $2 / 6=1 / 3$ of the house building work. Therefore, $1-1 / 3=$ $2 / 3$ of the house building work was done by Jeremy when working together with Flora. In other wordsJeremy spent 2 hours and completed $2 / 3$ of the house's building work, and thus every hour Jeremy can finish $1 / 3$ of the house. To finish the entire house, Jeremy needs $1 /(1 / 3)=3$ hours.
4. Kevin has a bucket of water. The bucket and the water weigh 13 kg in total. After you pour half of the water out, the bucket and the water weigh 8 kg . How much does the bucket weigh by itself?

Since the bucket and all of the water weighed 13 kg , and the bucket and half of the water weighs 8 kg , therefore half of the water weighs $13-8=5 \mathrm{~kg}$. Thus all of the water weighs 10 kg , making the bucket's weight the remaining 3 kg .
5. What is the smallest 7 digit number that is divisible by 11 ?

Since the smallest 7 digit number is 1000000 , thus the smallest 7 digit multiple of 11 must be $\frac{1000000}{11}$ rounded up times 11 , which is 1000010 .

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6. There is a deck of 52 cards, numbered 1 through 52. Alice is dealt 4 cards randomly, while Bob is dealt 1 . What is the probability that the largest of the numbers dealt went to Alice?

The probability of the largest card being any of the 5 dealt cards are equal. Since Alice was dealt 4 of the 5 dealt cards, the probability that the largest of the numbers dealt went to Alice is $\frac{4}{5}$
7. The sum of 5 consecutive numbers is $S$. If $x$ is the smallest number, what is the value of $S$ $-5 x$ ?

Let $n$ be the middle number. Thus the 5 consecutive numbers are $n-2, n-1, n, n+1, n$ +2 . So $S=(n-2)+(n-1)+n+(n+1)+(n+2)=5 n$. Since the smallest number is $n-$ 2 , we have that $x=n-2$. Thus $S-5 x=5 n-5(n-2)=10$. Thus the answer is 10 .
8. Some numbers are written on the blackboard. Each number is half of the sum of the rest of the numbers. How many numbers are written on the board?

There was a correction to this problem that stated that the numbers written on the board are positive (since any number of 0 s written on the blackboard would have this property). Working with a few examples you see that all the numbers have to be equal (if they aren't equal, then the smallest number of the list can't be equal to half the sum of the remaining numbers). Notice that if we have a list of 4 equal numbers, like $\{1,1,1,1\}$, or even a bigger list, then we won't have the required property. However, notice that the list $\{1,1$, $1\}$ has the required property (in fact any list of 3 equal numbers has the required property). Thus there are 3 numbers written on the board.

