

1. Alice owns 3 dogs, each of which weighs 35 pounds. After buying a cat, the average weight of her 4 pets becomes 30 pounds. How many pounds does Alice’s cat weigh?

Answer: 15 pounds

* *The total weight of Alice’s pet was $3 \times 35 = 105$ pounds. Let x represent the weight of her cat. We have $\frac{105 + x}{4} = 30$*

$$105 + x = 120$$

$$x = 15 \text{ pounds.}$$

2. $A + B + C + D = 36$, and $A - 2 = B + 2 = C \div 2 = D \times 2$. What is C?

Answer: 16

* *Let $k = A - 2 = B + 2 = C \div 2 = D \times 2$. So $A + B + C + D = 36$ says*

$$(k + 2) + (k - 2) + 2k + k/2 = 36$$

$$4k + k/2 = 36$$

$$9k = 72$$

$$k = 8$$

$$\text{Thus } C/2 = 8, C = 16.$$

3. A school has 6 periods and must teach math, gym, art, science, history, and English in these 6 periods. If math can only be taught in period 1, 2 or 3, and gym can only be taught in period 4, 5 or 6, how many ways are there to schedule the classes?

Answer: 216

* *We must assign a period number (1, 2, 3, 4, 5 or 6) to math, gym, art, science, history, and English. There are 3 ways to assign a number to math, and 3 ways to assign a number to gym, so 9 ways total to assign a number to math and gym (note math and gym do not fight for the same periods). Once math and gym are decided, there are $4! = 24$ ways to arrange art, science, history, and English. Thus there are $9 \times 24 = 216$ ways in total to schedule the morning.*

4. A circular pizza with diameter 10” costs \$20. If all pizzas were to have the same cost to area ratio, then what should be the cost of a square pizza with side length 12” (exact answer or rounded to 2 decimal places)?

$$\text{Answer: } \frac{576}{5\pi} \text{ or } \$36.67$$

* The cost to area ratio of the circular pizza is $\frac{20}{\pi(5)^2} = \frac{4}{5\pi}$. The area of the square pizza with side length 12 is 144. Thus the cost would need to be $\frac{4}{5\pi} \cdot 144 = \frac{576}{5\pi}$ or \$36.67.

Note: The answer using \$36.69 is also accepted; it is obtained by using 3.14 for π instead of more decimal places.

5. A triangle has integer side lengths and perimeter 24. If one side is twice as long as another side, what is the length of the third side?

Answer: 9

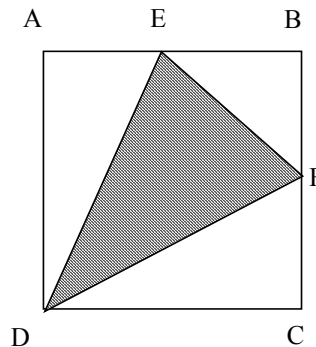
* Let x , $2x$, and y be the 3 side lengths. We have $x + 2x + y = 24$, or $3x + y = 24$. But x , y must be integers. Further more, the sum of two sides cannot be less than half the perimeter. So $3x > 12$ or $x > 4$. If $x \geq 6$ then $y \leq 6$, so $x + y = 12$, which cannot be since the sum of two sides is at least 12. We can check that $x = 5$, $2x = 10$, $y = 9$ works. Thus the length of the third side is 9.

6. A bucket filled with water has total weight 10 kg, and 99% of this total weight comes from water. After a while, some water evaporates so that only 98% of the total weight now comes from water. What is the total weight of the bucket of water now?

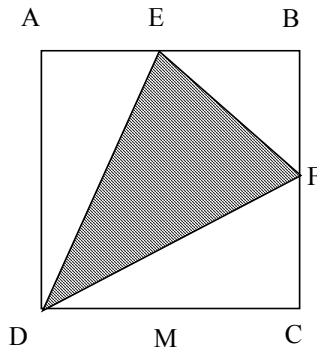
Answer: 5 kg

* Originally, the bucket weighs 10 kg total, with 99% of its weight coming from water. So the bucket weighs 0.1 kg. After the water evaporates, the bucket needs to weigh 2% of the total weight. So $(0.02) \cdot (\text{Total Weight}) = 0.1$ kg
Total weight = 5 kg.

7. In the figure, E is the midpoint of AB, and F is the midpoint of BC. What fraction of the entire square is shaded?

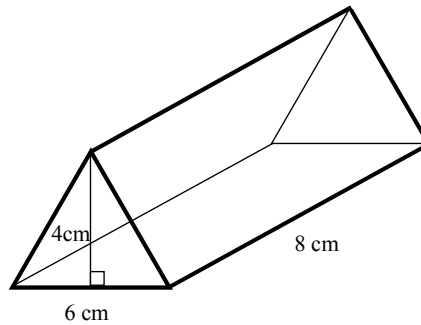


Answer: 3/8

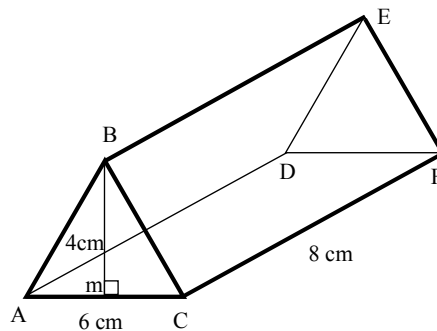


* Let's consider what fraction of the square isn't shaded. ADE is half of the half square (it is half of rectangle $AEMD$, where M is the midpoint of DC), so it is $\frac{1}{4}$ the square. Similarly CDF is $\frac{1}{4}$ the square. BEF is $\frac{1}{8}$ the square. So the unshaded fraction is $\frac{1}{4} + \frac{1}{4} + \frac{1}{8} = \frac{5}{8}$. So $\frac{3}{8}$ of the square is shaded.

8. In the diagram below, assuming the triangles are isosceles, what is the surface area of the triangular prism?



Answer: 152 cm^2



* Let m be the midpoint of AC . Since $Bm = 4 \text{ cm}$ and $mC = 3 \text{ cm}$, $BC = 5 \text{ cm}$, by the Pythagorean theorem.

$$\text{Area}(ABC) = \text{Area}(DEF) = \frac{1}{2}(6)(4) = 12 \text{ cm}^2$$

$$\text{Area}(ACFD) = 6 \cdot 4 = 48 \text{ cm}^2$$

$$\text{Area}(ABDE) = \text{Area}(CBEF) = 5 \cdot 8 = 40 \text{ cm}^2$$

$$\therefore \text{Total surface area is } 2(12) + 48 + 2(40) = 152 \text{ cm}^2$$