1. Julia has summed up all the odd numbers from 1 to 49 (including 1 and 49), and Darren has summed up all the even numbers from 1 to 49. By how much is Julia's sum larger than Darren's sum?

Solution: Consider any even number n that Darren adds to his sum. This even number must be between 1 and 49. It follows that n is at most 48 (since n is even). This means that n+1 is at most 49, and so n+1 is added in Julia's sum. It follows that for every number Darren adds to his sum, Julia adds a number one larger. The even numbers between 1 and 49 are 2x1, 2x2, 2x3, …, 2x24. Darren adds these numbers and Julia adds the numbers one larger than those. This means Julia's sum should be larger than Darren's by 24, as 24 numbers were used. However, Julia also adds the number 1 to her sum – this was not counted as Darren did not use the number 0. It follows that Julia's sum is larger than Darren's by 24+1=**25**.

2. Two 12-hour analog clocks are hanging beside each other. One of them always shows the correct time, but the other has had its hour hand and minute hand switched. Michael's dog barks whenever both clocks look identical. For example, the dog barks at 12:00 PM. How many times does the dog bark between 1:00 PM and 11:00 PM?

Solution: When the clocks look identical, it must be the case that their minute hands point to the same spot. However, one clock's minute hand actually represents the hours. This means that when the clocks show the same time, the minute hand of the correct clock points to the same spot as the hour hand of the correct clock. It follows that both hands of both clocks point to the same spot.
Therefore, to determine how many times the dog barks, it is enough to determine how many times the minute and hour hands of an ordinary clock overlap. It is clear that such an overlap must occur once between 1:00 PM and 2:00 PM (when the minute hand passes the hour hand). Similarly, such an overlap must occur once between 2 PM and 3 PM, once between 3 PM and 4 PM, and so on. In total, this overlap occurs **10** times between 1 PM and 11 PM.

3. Catherine has randomly chosen a number from the set {-4, -3, -2, -1, 1, 2}. Dan has also randomly chosen a number from that set (their chosen numbers might be the same). What is the probability that the product of their numbers is positive?

Solution: Their product is positive either if they both got positive numbers or if they both got negative numbers. The probability of getting a positive number is 2/6=1/3, since there are 2 positive numbers out of 6 total numbers. Therefore, the probability that both Catherine and Dan got positive numbers is (1/3)x(1/3)=1/9. The probability of getting a negative number is 4/6=2/3, since there are 4 negative numbers out of 6 total numbers. Therefore, the probability that both Catherine and Dan got negative numbers is (2/3)x(2/3)=4/9. In total, the probability that the product is positive must be 1/9+4/9=**5/9**.

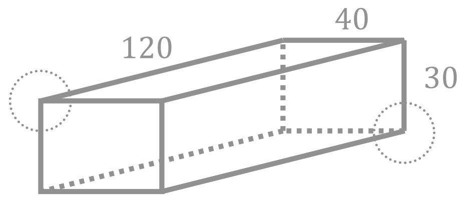
4. If I were to multiply the two numbers 1,234,567,890,987,654,321 and 113,355,779,986,420, what would be the last three digits of the result?

Solution: We can write
1,234,567,890,987,654,321=1,234,567,890,987,654x1000 + 321, and
113,355,779,986,420=113,355,779,986x1000 + 420.
We are then looking for the last 3 digits of
(1,234,567,890,987,654x1000 + 321)x(113,355,779,986x1000 + 420).
If we expand the above product, we can notice that anything multiplied by 1000 cannot affect the last 3 digits. This means it is safe to ignore anything multiplied by 1000. In other words, the 3 digits we are looking for are the last three digits of 321x420. Since 321x420=134,820, the 3 digits we are looking for are **820**.

5. Change only one digit of 12110 so that the number is divisible by 225. Write down the resulting number.

Solution: Notice that 225=25x9. Therefore we must make the result divisible by 9 and by 25. A number is divisible by 25 if and only if its last two digits are 00, 25, 50, or 75. Since the last two digits of 12110 are 10, we must change the 1 to either a 0 or a 5. Hence the resulting number should either by 12100 or 12150. The sum of the digits of 12100 is 4, while the sum of the digits of 12150 is 9. This means that 12100 is not divisible by 9, while 12150 is divisible by 9. Therefore **12150** is the answer.

6. A box has width 40cm, height 30cm, and length 120cm. What is the distance between the two farthest corners of the box?



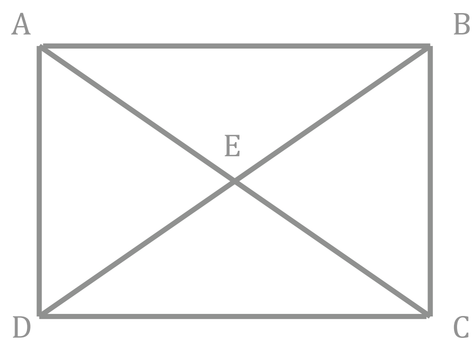
Solution: Consider the two marked corners and the corner between the "120" label and the "40" label. These three corners form a triangle. This triangle goes through the interior of the box. It is a right triangle, and we are asked for the length of its hypotenuse (since that is the distance between the two marked corners). We want to use the Pythagorean theorem. To do this, we must know the lengths of the other sides of this triangle.

One of the sides has length 120. The other side is the diagonal of a 30 by 40 rectangle. Using the Pythagorean theorem, the length of this side must be the square root of $30^2+40^2=900+1600=2500$, which is 50. Therefore, the lengths of the sides are 120 and 50. Using the Pythagorean theorem again, the desired length of the hypotenuse is equal to the square root of $50^2+120^2=2500+14400=16900$, which is 130. We conclude that the distance between the two farthest corners is **130cm**.

7. What is the greatest number that is a factor of both 7988 and 8978?

Solution: Any number that is a factor of both 7988 and 8978 must be a factor of their difference. We have 8978-7988=990, which factors as 2x3x3x5x11. Using the divisibility rules for 2, 3, 5, and 11, it is easy to see that 7988 is divisible by 2, but not by 3, 5, and 11. It follows that the largest possible common factor is 2, and indeed 2 is a factor of both 7988 and 8978. Therefore the answer is **2**.

8. In the diagram below, ABCD is a rectangle with perimeter 60. Given that the perimeter of triangle ABC is 12 more than the perimeter of triangle ABE, what is the length of AB?



Solution: Notice that the perimeter of triangle ABC is the sum of the lengths of AB, AE, EC, and BC. The perimeter of triangle ABE is the sum of the lengths of AB, AE, and BE. Since ABCD is a rectangle, its diagonals are equal, and they bisect each other. This means that BE has the same length as EC. It follows that the difference between the perimeter of ABC and the perimeter of ABE is just the length of BC. Since this difference is 12, the length of BC is 12. This means the length of AD is also 12. Since the perimeter of ABCD is 60, we have 12+|AB|+12+|CD|=60, so |AB|+|CD|=36. Since |AB| is the same length as |CD|, its length must equal 36/2=**18**.