1. The sum of the ages of Alice and Bob is 78. Bob is 32 years older than Alice. How old is Alice?

   Solution: Bob's age is equal to Alice's age plus 32. This means the sum of the ages of Alice and Bob is equal to twice Alice's age plus 32. This is 78, so twice Alice's age must equal 78-32=46. Therefore Alice is 46/2=**23** years old.

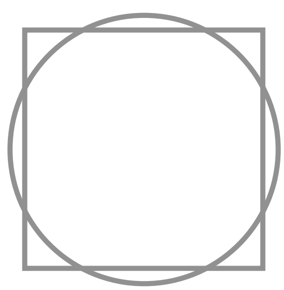
2. How many non-zero digits are there in 2000^{12}?

   Solution: If you know your exponentiation rules, you can write
   $2000^{12}=(2\times 1000)^{12}=(2^{12})\times(1000^{12})=4096\times(10^{3\times 12})=4096\times 10^{36}$.
   Multiplying by a power of 10 only adds zeros at the end of the number, so we can tell that the are only **3** non-zero digits (they are 4, 9, and 6).

3. Two 12-hour, analog clocks are hanging on the wall beside each other. One of them shows the correct time and the other is running backwards. At noon, both clocks show 12:00 at the same time. When will both clocks show the same time next?

   Solution: In order for both clocks to show the same time, their hour hands must point at the same place in the clock. The hour hands both start at the top of the clock (at 12), but one moves to the right (clockwise) and the other moves to the left (counter-clockwise). This means the hour hands will next be at the same place when they both point at 6. Moreover, when the hour hands point to exactly 6, the minute hands both point to exactly 12, so the clocks show the same time. This happens at **6pm**.

4. A circle and a square have the same centre. The circle intersects the square twice on each of its four sides. The area inside the square but outside the circle is equal to the area inside the circle but outside the square. If the side length of the square is 2, what is the area of the circle?



   Solution: The area of the square is 4. If start with a square, remove the part of the square that is outside of the circle, and then add the part of the circle that is out of the square, we end up with the circle. Since the area of the region we removed from the square is equal to the area of the region we added, we conclude that the area of the circle is **4**.

5. How many numbers between 1 and 300 (including 1 and 300) are divisible by both 4 and 6?

Solution: A number is divisible by both 4 and 6 if and only if it is divisible by 12. Therefore, we simply need to count the numbers divisible by 12 between 1 and 300. The numbers divisible by 12 in that region are 1x12, 2x12, 3x12, …, 25x12, since 25x12=300. This means the answer is **25**.

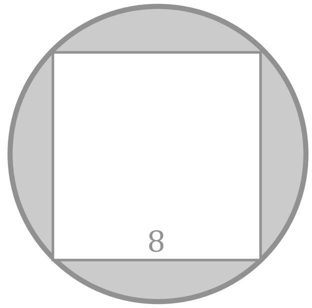
6. George is taking a bus to school. The chance that it rains is 1/4. The chance he misses his bus is 1/3, regardless of whether it is raining. What is the chance he misses the bus when it is raining?

Solution: Consider 12 different days in which George goes to school. On average, it rains on 1/4 of those days – so it rains on 3 days on average. On each of those days, George has a 1/3 chance of missing his bus. This means that on average, George misses his bus on 1 of those 3 rainy days. Therefore, out of the 12 days, George can expect to miss his bus in the rain 1 day. We conclude that his chance of missing the bus in the rain is **1/12**.

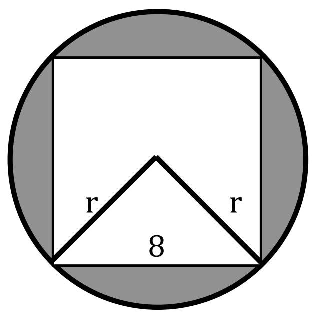
7. Harry must pick an even number n with the property that both n/2 and 4xn are three-digit numbers. How many choices does he have?

Solution: Since n/2 must be a 3-digit number, it must be at least 100. This means n must be at least 2x100=200. Since 4xn must be a 3-digit number, it must be smaller than 1000. This means n must be smaller than 1000/4=250, so n is at most 249. We also require that n be even. The even numbers between 200 and 249 are
200+2x0, 200+2x1, …, 200+2x24. There are 25 numbers in this list, so there are **25** choices for n.

8. A square with side length 8 is inscribed in a circle in the figure below. Find the area of the shaded region.



Solution: Drawing the radii of the circle to the corners of the square, we get the picture



where r is the radius of the circle. The area of the square is 8x8=64. The area of the triangle is a quarter of that, so the area of the triangle is 64/4=16. Notice that since the triangle is a right-angled triangle, its area can also be calculated by multiplying the lengths of the two sides of length r and dividing by 2 (using the base times height over 2 formula). This means that rxr/2=16, so r^2=32. The area of the circle is therefore 32xpi. To get the area of the shaded region, we just subtract the area of the square. This means the area of the shaded region is **32xpi-64**.