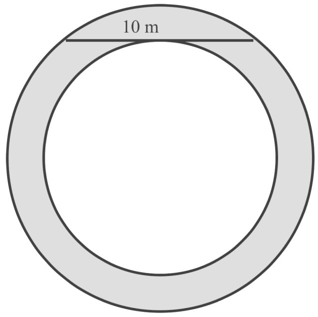
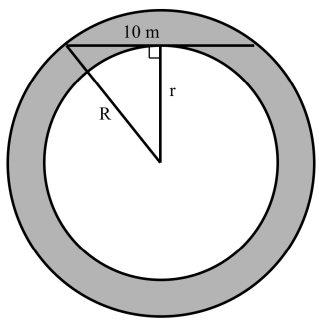
1. A circular path (shaded) surrounds a circular lake. The longest straight-line distance across the path is 10 metres. What is the total area of the path?

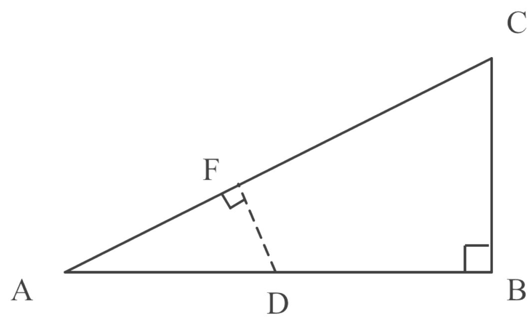


Solution: The best way to solve this problem is to assign variables to the radii of the lake and the lake with the path. Let r be the radius of the lake, and let R be the radius of the lake with the path. We draw them as follows.



By the Pythagorean theorem (勾股定理), we have $R^2 - r^2 = 5^2 = 25$. Also, The area of a circle with radius R is πR^2 and the area of a circle with radius r is πr^2. Therefore the area of the path is $\pi R^2 - \pi r^2 = \pi(R^2 - r^2) = 25\pi$, so the answer is **25pi**.

2. ABC is a right triangle with the right angle at B. D is the midpoint of AB, and F is chosen on AC such that DF is perpendicular to AC. If |AB|=2 and |BC|=1, find the area of triangle ADF.



Solution: Since D is the midpoint of AB, we have |AD|=1. Also, by the Pythagorean theorem (勾股定理), $|AC| = \sqrt{2^2 + 1^1} = \sqrt{5}$. Notice that ABC and ADF are right triangles that share an angle at A. Therefore they are similar, with ABC~AFD. AC corresponds to AD, so the ratio between ABC to AFD is $\sqrt{5}:1$. This means that the ratio of their areas is 5:1. Since the area of ABC is 2x1÷2=1, the area of ADF must be **1/5**=**0.2**.

3. At what time between 2 P.M. and 3 P.M. do the minute hand and the hour hand of a clock make an angle of 180° with each other? Round your answer to the nearest minute.

Solution: Since the time is between 2 and 3, the hour hand is between 2 and 3. The minute hand must be 180 degrees away, so it must be between 8 and 9. This means the time must be between 2:40 and 2:45. Notice that when the minute hand makes a full cycle around the clock, the hour hand only makes one-twelfth of a cycle, So the hour hand moves 12 times slower than the minute hand. Let’s measure everything in terms of the “minute” marks on the clock. If the minute hand moves x minutes, the hour hand moves x/12 minutes. Also, the hour hand starts at 2pm, which is at 10 minutes, and we know the two hands are 30 minutes apart. If the minute hand is at x minutes in the solution, then we have x-(10+x÷12)=30, or 12x-120-x=360, so 11x=480. This means x=43.6, which is 44 to the nearest minute. This means the time is **2:44 PM**.

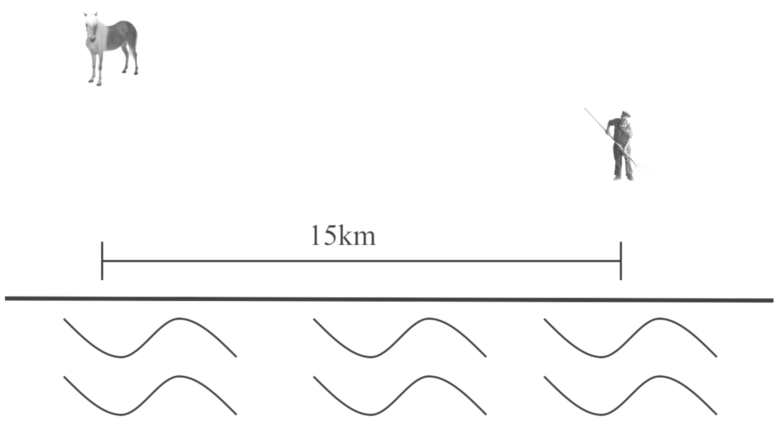
4. You shuffle a deck of 54 cards (standard 52 plus the two jokers), and then you draw cards from the deck one at a time until you finish drawing the whole deck. What is the probability that you draw the two jokers before any of the four aces?

Solution: We can ignore all the cards in the deck except the two jokers and the four aces. Imagine the deck only has 6 cards: the two jokers and the four aces. The only way to draw jokers before aces is if the deck is Joker, Joker, Ace, Ace, Ace, Ace. However, there are 6 choose 2 ways to arrange the 6 cards (we treat the aces as identical to each other, and the jokers as identical to each other). Since 6 choose 2 is 6x5÷2=15 and there is only one way to arrange the jokers before the aces, the probability must be **1/15**.

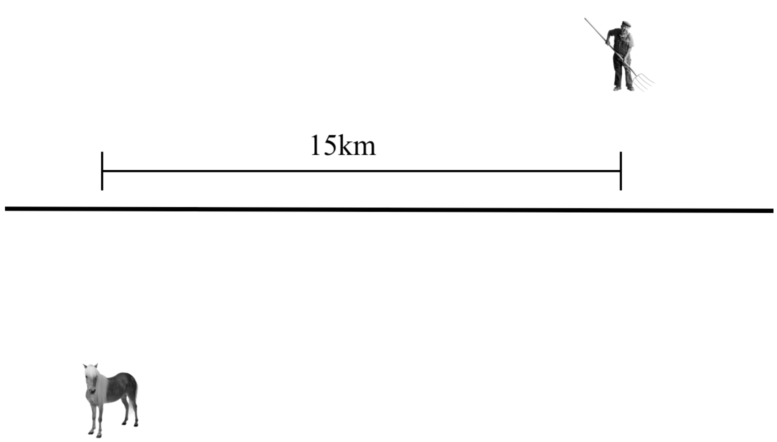
5. What is the sum of the digits of all integers from 1 to 100?

Solution: The sum of all the units digits of the integers from 1 to 10 is 1+2+3+…+9=45. The sum of all the units digits of the integers from 10 to 20 is again 1+2+3+…+9=45. Similarly, the sum of all the units digits from 20 to 30, is 45, and so on. Therefore the sum of all the units digits from 1 to 100 is just 10x45=450.
Now we consider the tens digits. The digit 1 appears ten times as a tens digit: in 10, 11, 12, 13, 14, 15, 16, 17, 18, and 19. Similarly, the digits 2, 3, 4, 5, 6, 7, 8, and 9 also appear ten times as a tens digit. This means the sum of all tens digits is
10x1+10x2+10x3+…+10x9=10x(1+2+3+…+9)=10x45=450.
Finally, there is only 1 hundreds digit, in 100. Therefore the sum of the hundreds digits is 1. Putting this all together, the sum of all the digits of the integers from 1 to 100 is 450+450+1=**901**.

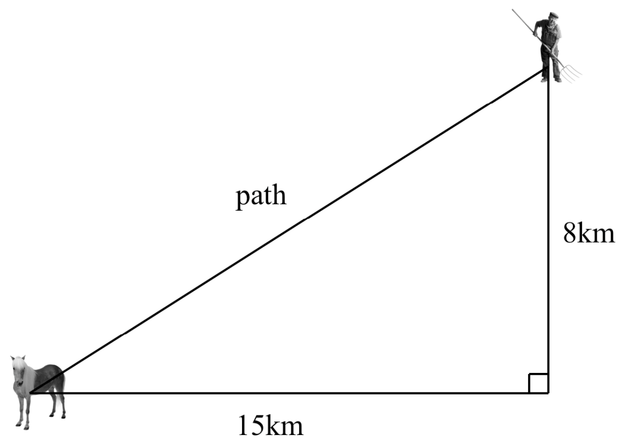
6. A farmer stands 3km away from a straight river. His horse is standing 5km away from the river, 15km downstream from the farmer. The farmer wishes to get some water from the river and bring it to his horse. What is the shortest distance the farmer must travel? (Drawing not to scale).



Solution: The best way to solve the problem is to imagine that the horse is on the other side of the river. Since the farmer is required to go to the river before going to the horse, these problems are the same. Then we have the diagram



The shortest path to the horse passing through the river is now a straight line (because the shortest path between any two points is a straight line). The horse and the farmer are 15km apart along the river, and they are 3+5=8km apart perpendicular to the river. We have the diagram



By the Pythagorean theorem, the length of the path is $\sqrt{15^2+8^2}=\sqrt{225+64}=\sqrt{289}=17$ so the answer is **17**.

7. A man takes the train from work every day and arrives at the station at precisely 6pm, at which time his wife arrives at the station to pick him up and drive him home. One day, he takes an earlier train and arrives at the station at 5pm, and he starts to walk home. He meets his wife at some point on the way home, where she picks him up and they drive home, arriving 20 minutes earlier than usual. For how long has the man been walking home from the station?

Solution: Since they arrived home 20 minutes earlier than usual but the wife left at the same time as usual, she must have driven 20 minutes less than usual. The wife avoided driving back and forth the distance that the husband walked. This means that the distance the husband walked would take the wife 10 minutes to drive. In particular, when the wife met the husband, she was 10 minutes away from the train station. Since she would have arrived at the train station at 6, it must have been 10 minutes before 6 when they met. This means the husband walked from 5 PM until 10 minutes before 6 PM, so he walked for **50 minutes**.

8. A right-angled triangle has all integer side lengths, and one of the sides has length 11. What is its perimeter?

Solution: Suppose the lengths of the other two sides are x and y, where x and y are integers. If 11 is the length of the hypotenuse, then by the Pythagorean theorem (勾股定理), $x^2 + y^2 = 11^2 = 121$, so $x^2 = 121 - y^2$. Notice that y<11 and y is an integer. However, 121-1=120, 121-4=117, 121-9=112, 121-16=105, 121-25=96, 121-36=85, 121-49=72, 121-64=57, 121-81=40, and 121-100=21. None of these numbers are squares, which is a contradiction, because $x^2 = 121 - y^2$. This means 11 is not the length of the hypotenuse.

Let x be the length of the hypotenuse, and y the length of the remaining side. Then by the Pythagorean theorem (勾股定理), $x^2 = 121 + y^2$. Rearranging, we have $x^2 - y^2 = 121$, so $(x - y)(x + y) = 121$. Since x-y and x+y are integers, they are factors of 121. But the only factors of 121 are 1, 11, and 121. Since x+y>x-y, we must have x+y=121 and x-y=1. Adding these two equations gives 2x=122, so x=61. This means 61+y=121, so y=60. The perimeter of the triangle is therefore 61+60+11=**132**.