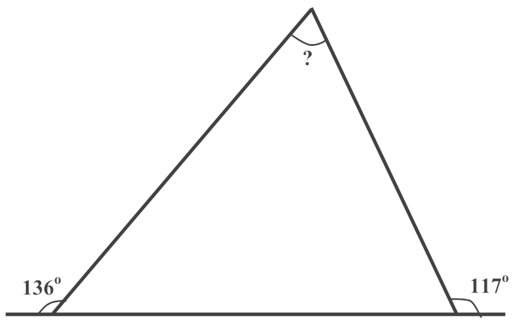
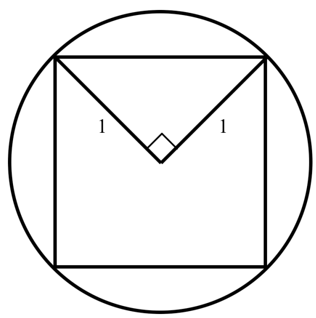
1. Determine the angle in the diagram marked by "?".



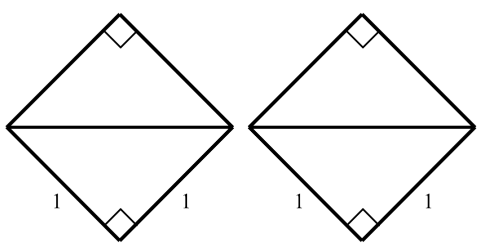
Solution: The angle inside the triangle next to "136" is 180-136=44. The angle inside the triangle next to "117" is 180-117=63. Since the sum of the angles in a triangle is 180, we must have 44+63+?=180, so 107+?=180, which means ?=**73**.

2. Consider the largest square that can fit into a circle of radius 1. What is the area of this square?

Solution: The first thing to do is to draw the following diagram:



Notice that the square inside the circle can be split up into 4 right triangles that are identical to the one in the diagram. This triangles can be put together to make two 1 by 1 squares:



Since the area of each the two squares is 1x1=1, the area of both squares is 1+1=2. Thus the area of the original, large square is **2**.

3. 3 apples weigh the same as an orange, but 2 pears weigh less than an apple. A banana and 4 pears together weigh more than 2 oranges, but a banana weighs less than the sum of the weights of one orange, one apple and one pear. What is the heaviest fruit?

Solution: First of all, 3 apples weigh the same as an orange, so an apple weighs less than an orange. Also, 2 pears weigh less than an apple, so an apple weighs more than a pear. This means that orange > apple > pear. Banana is trickier. We know that banana + 4 pears > 2 oranges, so banana > 2 oranges – 4 pears. Notice that 4 pears weigh less than 2 apples and 2 apples weigh less than an orange. This means that 2 oranges – 4 pears > 1 orange, so banana > orange. Therefore **banana** is the heaviest fruit. Notice that the last piece of information (banana<orange+apple+pear) is not needed.

4. Find the largest number that is a factor of both 1014 and 1027.

Solution: If a number is a factor of both 1014 and 1027, then it must also be a factor of 1027-1014=13, so it is either 1 or 13 (since 13 is prime). Both 1014 and 1027 are divisible by 13, so the largest common factor is **13**.

5. Kate tosses one coin and Larry tosses two coins. What is the probability that Kate and Larry get the same number of heads?

Solution: The probability that Kate gets 1 heads is 0.5. There are 4 possible outcomes for Larry's coins: HH, HT, TH, and TT. Two of these outcomes result in one heads, so the probability of Larry getting 1 heads is 2/4=0.5. Therefore the probability of both Kate and Larry getting 1 heads is 0.5x0.5=0.25. The probability of Kate getting 0 heads is 0.5, and the probability of Larry getting 0 heads is 1/4=0.25. The probability of both Larry and Kate getting 0 heads is therefore 0.5x0.25=0.125. Finally, the probability of Kate and Larry getting the same number of heads is equal to the probability they get 1 heads plus the probability they get 0 heads, so it is 0.25+0.125=**0.375** =**3/8**.

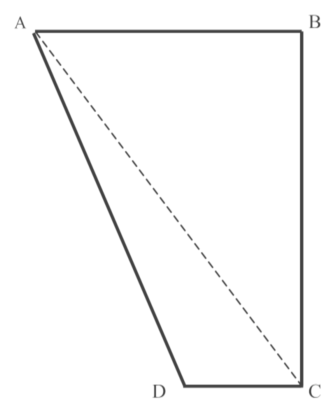
6. A number is said to be a palindrome if, when read from right to left, it is the same as when it is read from left to right (e.g. 101, 999). How many palindromes are there between 100 and 1000?

Solution: Since 1000 is not a palindrome, the answer needed is the same as number of 3-digit palindromes. There are 9 possibilities for the first digit of a palindrome (1 to 9), and 10 possibilities for the second digit (0 to 9). For the third digit, there is only 1 possibility: the third digit of the palindrome must be equal to the first digit. Therefore the number of 3-digit palindromes is 9x10x1=**90**.

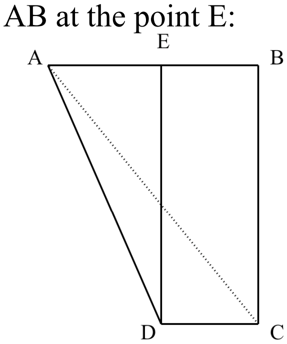
7. Mary and Nicholas are waiting to take different buses home. Their bus stops are on the same street, 2.1km apart. Mary walks at 50 metres per minute, and Nicholas walks at 150 metres per minute. Nicholas's bus comes 10 minutes later than Mary's bus. They want to separate at a point on the street such that they can stay together for as long as possible and still both catch their buses. How many minutes are there from the time they leave each other to the time Mary steps on her bus?

Solution: One of the best ways to solve the problem is to imagine that time is going backwards. Then Nicholas's bus comes 10 minutes *before* Mary's bus, and they want to meet each other as soon as possible. Their best strategy is simply to walk towards each other as fast as they can. Nicholas starts 10 minutes earlier than Mary and walks at 150 metres per minute, so he walks 10x150=1500 metres. Since their bus stops are 2100 metres apart, he is 2100-1500=600 metres away from Mary's stop at the end of the 10 minutes. Then Mary steps out of her bus, and starts walking towards Nicholas at 50 metres per minute. This means that Mary and Nicholas are approaching each other at 150+50=200 metres per minute, and they are 600 metres apart, so they must walk for 600÷200=**3** minutes between the time Mary steps on her bus and when they meet.

8. ABCD is a trapezoid with right angles at B and at C. If |AB|=9, |CD|=4, and |AD|=13, find |AC|. (Note: |AB| is the length of the line segment AB).



Solution: First, we find |BC|. Let's draw a new line in the diagram, passing through D and perpendicular to AB, meeting AB at the point E:



Then |BC|=|DE| and |BE|=|CD|=4, so |AE|=|AB|-|BE|=9-4=5. Also, ADE is a right

triangle, so by the Pythagorean theorem,
$|DE|^2 = |AD|^2 - |AE|^2 = 13^2 - 5^2 = 169 - 25 = 144$.
This means $|BC|^2 = 144$. Also, ABC is a right triangle, so by the Pythagorean theorem (勾股定理), we have $|AC|^2 = |AB|^2 + |BC|^2 = 9^2 + 144 = 81 + 144 = 225$, so |AC|=**15**.