1. There are 8 people in a classroom. Each person has to shake hands once with everyone else in the room. How many handshakes would occur?

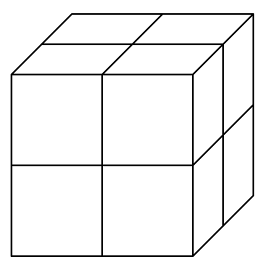
   Solution: There are 8 people in the room, and each of them shakes hands with everyone else. This means each of the 8 people shakes hands with 7 people, so each of the 8 people shakes their hand 7 times. This means there are 8x7=56 hands being shaken. Each handshake involves 2 different hands, so there were 56÷2=**28** handshakes.

2. What is the smallest positive number divisible by 2, 3, 4, 5, and 6?

   Solution: The best way to do this is to look at the prime factors of the answer. The prime factor 2 must occur twice since 4 contains the prime 2 twice. Also 3 must occur once, and 5 must occur once. The answer is therefore 2x2x3x5=**60**.

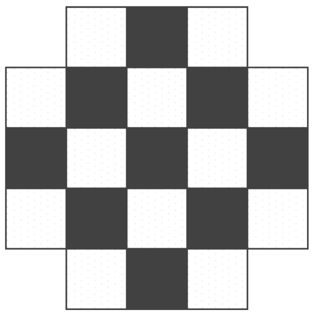
3. A cube is cut up into 8 identical smaller cubes. If the height of the original cube is 6, what is the height of the new cubes?

   Solution: One way to solve this question is to visualize a cube cut into 8 identical smaller cubes. It may look like

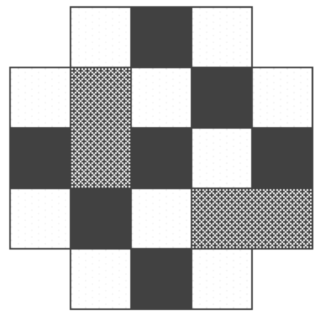


   Then it is clear that the height of the smaller cubes is half the height of the large cube, so the answer is **3**. (Another way to solve this problem is to realize that the volume of each small cube is one-eighth the volume of the large cube, and that the volume of a cube is the height raised to the third power.)

4. Jimmy has a 5x5 chessboard with its 4 corners removed. He wants to put dominoes on the board such that they do not overlap. One domino has the size of 2 squares on the chessboard, which means each domino covers exactly two adjacent squares: one white and one black. What is the maximum number of dominoes he can place on this chessboard at the same time?

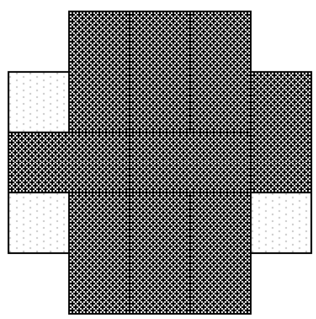


The chessboard



Example of two dominoes on the chessboard

Solution: Each domino covers one black square and one white square, and there are 9 black squares on the chessboard. This means there can be at most 9 dominoes on the board at once. Also, it is possible to place 9 dominoes on the board. There are several ways to do this, but here is one of them:



The answer is **9**.

5. The perimeter of a rectangle is 30 cm. The width of the rectangle is half of its length. Find the area of the rectangle.

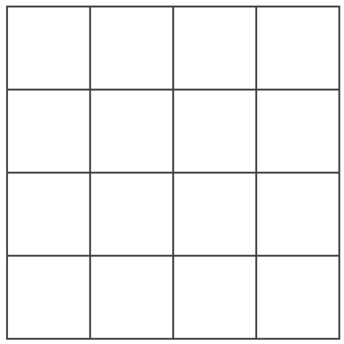
Solution: We know that the length is twice the width. Also, we know that length+length+width+width=30cm. Since length is 2 times width, then length+length+width+width= 6 times width. This means that 6 times width is 30cm, so width=30/6=5 cm. Then length=2 times width=10cm. The area is 5x10=**50**.

6. Suppose one orange weighs the same as 2 peaches, and one peach weighs the same as 3 apricots. How many apricots weigh the same as 2 oranges and 1 peach together?

Solution: One orange weighs the same as 2 peaches, which is the same as 2x3=6 apricots. This means that 2 oranges and 1 peach weigh the same as 6+6+3=**15** apricots.

7. A 4x4 grid is filled with numbers, with the following properties:
   a. The sum of the 4 numbers in the first row is 15.
   b. The sum of the 4 numbers in the second row is 26.
   c. The sum of the 4 numbers in the third row is 35.
   d. The sum of the 4 numbers in the fourth row is 44.
   e. The sum of the 4 numbers in each column is the "magic number".

   What is the magic number?



Solution: The sum of all the numbers in the grid must be 15+26+35+44=120. Since the numbers in each column sum up to the magic number, it must be that 4 times the magic number is also equal to the sum of all the numbers in the grid. This means 4 times the magic number equals 120, so the magic number is 120÷4=**30**.

8. How many pairs of prime numbers add up to 199?

Solution: All prime numbers other than 2 are odd, and a sum of two odd numbers is even. Since 199 is odd, it is not a sum of two odd primes. This means the only possible way to write 199 as a sum of primes is 2+197. We need to check that 197 is prime, and it is, because it is not divisible by 2, 3, 5, 7, 11, and 13 (we don't need to check further, because we only need to check primes up to the square root of 197, which is less than 17). Therefore, the only pair of prime numbers that sums up to 199 is the pair 2 and 197, so there is one **1** pair.